1)

T = 28000 K

R = 5,16 x10^9 m

$* \xrightarrow{d = 123 pc} \bullet$

$\sigma = 5,67 \times 10^{-8}\ W.m^{-2} K^{-4}$

$\frac{2.5}{2.8}$

a)

$L = A \cdot F = 4\pi R^2 \cdot \sigma T^4 = 4\pi (5,16 \times 10^9)^2 \cdot \sigma \cdot (28000)^4 =$

$= 4\pi \cdot 2,66 \times 10^{19} \cdot 5,67 \times 10^{-8} \cdot 6,14656 \times 10^{17}$

$= 1,17 \times 10^{31}$ Watts //

b) Comparando com o sol

$M - M_\odot = -2,5 \log L/L_\odot \rightarrow M = M_\odot - 2,5 \log L/L_\odot$

$M = 4,85 - 2,5 \log (1,17 \times 10^{31} / 3,846 \times 10^{26}) = 4,85 - 11,21 \approx -6,36$ mag //

use M_sol p/ diferenciar de massa solar ($M_\odot$)

c) $m = M + 5 \log d/10 = -6,36 + 5 \log \frac{123}{10} = -6,36 + 5,45 \cong -0,91$

$\approx -1,0$

d) $m - M = 5 \log d/10 = 5 \cdot 1,09 = 5,45$

$\hookrightarrow m - M = -1 + 6,36 = 5,36$ //

próximos OK!

e) Fluxo sup $= \frac{L}{4\pi R^2} = \frac{1,17 \times 10^{31}}{4\pi (5,16 \times 10^9)^2} = \frac{1,17 \times 10^{31}}{3,35 \times 10^{20}} = 3,50 \times 10^{10}$ watt/m²

f) $d = 123 \text{ pc} = 3{,}086\times10^{16} \text{ m} \cdot 8$ $\quad = 3{,}76\times10^{18}$ $\quad = 1\text{pc}$

X

6.47×10^{-8} W/m²

$$F = \frac{L}{4\pi d^2} = \frac{1{,}17\times10^{31}}{4\pi(\underbrace{3{,}086\times10^{16}}_{x})^2} = \frac{1{,}17\times10^{31}}{1{,}2\times10^{34}} = 9{,}78\times10^{-4} \text{ watt/m}^2$$

X

Sol = 1365 watt/m² ⟶ Fluxo sol > Fluxo $_*$

QUANTO? $\quad F_* = 4{,}7\times10^{-11} F_\odot$

g)

$$\lambda max = \frac{2900}{T} = \frac{2900}{22000 \to 29000} = 0{,}10 \ \mu m$$

✓

ou

$$\lambda max = \frac{0{,}2898}{T} = 1{,}04\times10^{-5} \text{ cm} = 104 \text{ nm}$$

2)

a) Lei de Rayleigh-Jeans $\quad \frac{1.2}{1.2}$

$$B_\lambda(T) = \frac{2cKT}{\lambda^4}$$

→ Planck

→ arrumar

$$\lim_{\lambda \to hc/KT} \frac{2hc^2/\lambda^5}{e^{hc/\lambda KT} - 1} = \frac{2hc^2 K^5 T^5}{h^5 c^5 \cdot (e-1)} = \frac{2(KT)^5}{h^4 c^3 (e-1)} = \frac{2KT \cdot c}{\underbrace{(hc/KT)^4}_{\lambda} \underbrace{(e-1)}_{\approx 1}}$$

$$= \frac{2cKT}{\lambda^4} = B_\lambda(T)$$

3) $\mu = 10,3577''/ano$ $\lambda = 656,034\,nm$

$p = 0,594901''$ $\lambda_0 = 656,281\,nm$

$\underbrace{c = 3\times10^8\,m/s \Rightarrow 299792\,km/s}$

1.0 / 1.2

a)

$$\frac{\lambda-\lambda_0}{\lambda_0} = \frac{v}{c} \rightarrow v_r = \frac{c(\lambda-\lambda_0)}{\lambda_0} = \underset{m/s}{3\times10^8}\frac{(656,034 - 656,281)}{656,281}$$

$$v_r = \overset{\rightarrow km/s}{299792}\cdot(-3,76\times10^{-4}) = -112,7\,km/s \,// \text{ ou } = -1,12\times10^{5}\,m/s$$

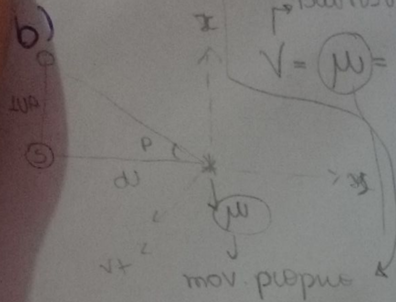
$\sim 113\,Km/s$

b) $V = \mu = 10,3577''/ano$

transversal

mov. próprio

ok



c) $v_{tot}^2 = v_t^2 + v_r^2 \Rightarrow km/s$

tangencial

CUIDADO!

89,4 Km/s

$$v_t = 4,74\cdot\frac{\mu(''/ano)}{p('')}(km/s) = 4,74\cdot\frac{10,3577}{\underline{0,594901}} = 83,05\,km/s \,//$$

0.54901 ≠ DADO DO PROBLEMA

$\therefore$

$$V_{TOT} = \sqrt{-112,7^2 + 83,05^2} = 139,9\,km/s \approx 1,4\times10^5\,m/s \,//$$

144 Km/s

4)

0.2 / 0.4

$$E = \frac{hc}{\lambda} = \frac{\cancel{1200}\ eVnm}{10\cancel{0}\ nm} = 12eV$$

1240   12,4

8.4

$$W = \underbrace{\Delta K}_{K_{MAX} - \cancel{K_0}} = E_{FOTON} - \phi \Rightarrow 12eV - 4\,eV = 8eV \,//$$

5) $P = 49{,}94$ anos

$p'' = (0{,}37921 \pm 0{,}00158)''$

$a = 7{,}61''$

$a_A / a_B = 0{,}4666$

$d = 1/p'' = 1/0{,}37921 =$

$\downarrow$ (pc)

$\frac{1{,}0}{1{,}2}$

a)

$$\frac{m_1}{m_2} = \frac{r_2}{r_1} \longrightarrow \frac{m_B}{m_A} = \frac{a_A}{a_B} = 0{,}4666$$

$$\therefore m_B = 0{,}4666\, m_A \approx 0{,}47$$

$$a = \theta''/p'' = 7{,}61''/0{,}37921'' = 20{,}06$$

$$P^2(\text{Anos}) = \frac{a^3\,(\text{UA})}{[m_A + m_b]} = \frac{20{,}06^3}{[m_A + 0{,}47 m_a]} = 49{,}94^2$$

$$1{,}47 m_a = \frac{20{,}06^3}{49{,}94^2} = 3{,}24 \longrightarrow m_a = 3{,}24/1{,}47 = 2{,}20\ M_\odot$$

$$\begin{cases} m_a \cong 2{,}20\ M_\odot \\ m_b \cong 1{,}03\ M_\odot \end{cases}$$

b) magnitudes absolutas?

$$\begin{cases} M_{SA} = 1{,}36 \\ M_{Sb} = 8{,}79 \\ M_\odot = 4{,}85 \end{cases}$$

Temos (comparar c/ sol)

$$M - M_\odot = -2{,}5 \log (L/L_\odot) \therefore$$

$$\frac{L}{L_\odot} = 100^{(M_\odot - M)/5}$$

Sirius A

$$\frac{L_A}{L_\odot} = 100^{(4{,}85-1{,}36)/5} = 100^{3{,}49/5} = 100^{0{,}698} \approx 24{,}9\ L_\odot$$

Sirius B

$$L_B = 100^{(4{,}85-8{,}79)/5} = 100^{-3{,}94/5} = 100^{-0{,}788} \approx 0{,}026\ L_\odot$$

$$\begin{cases} L_A = 9{,}5 \times 10^{27}\ \text{watt} \\ L_b = 9{,}98 \times 10^{24}\ \text{watt} \end{cases}$$

c) $F = L/4\pi R^2 \rightarrow \sigma T^4 = L/4\pi R^2 \rightarrow R^2 = L/4\pi\sigma T^4$

$R_b^2 = \dfrac{L_b}{4\pi\sigma T_b^4} = \dfrac{9{,}98\times10^{24}}{4\pi.(5{,}67\times10^{-8}).(24790)^4} = \dfrac{9{,}98\times10^{24}}{2{,}69\times10^{11}} = 3{,}71\times10^{13}$ ✓ ✗

$\rightarrow R_b = \sqrt{3{,}71\times10^{13}} \cong 6\times10^6\,m$ //

QUANTO MAIOR?

Então

$R_{sol} = 6{,}96\times10^8\,m$ → É bem maior que o raio de Sirius B

$R_{Terra} = 6{,}37\times10^6\,m$ → O raio de Sirius B é bem próximo da Terra, e menor

QUANTO?

6) $P = 6{,}31\,anos = 1{,}99e8\,s$ | $(t_b - t_a) = 0{,}58$ dias $= 50112\,s$

v_{max} radial | $(t_c - t_b) = 0{,}64$ dias $= 55296\,s$

$\hookrightarrow v_{mA} = 5{,}4\,Km/s = 5400\,m/s$ | $m_{max} = 5{,}4$

$v_{mB} = 22{,}4\,Km/s = 22400\,m/s$ | $m_{minp} = 9{,}2$ } mag $e=0$!

$i = 90°$ | $m_{mins} = 5{,}44$

$\dfrac{1.6}{1.6}$

a) Razões de massas

$\dfrac{m_1}{m_2} = \dfrac{v_{r2}}{v_{r1}} \Rightarrow \dfrac{m_A}{m_b} = \dfrac{v_{rb}}{v_{rA}} = \dfrac{22{,}4}{5{,}4} = 4{,}15$ // ✓

b) $i = 90°$! Soma das massas

$m_a + m_b = \dfrac{P}{2\pi G}\dfrac{v_{ar}^3}{\underbrace{sen^3 90°}_{1}}\left(1 + \dfrac{v_{br}}{v_{ar}}\right)^3 = \dfrac{P}{2\pi G}\left(v_{ar} + v_{br}\right)^3 = \dfrac{1{,}99\times10^8}{2\pi(6{,}67\times10^{-11})}(5400+22400)^3$

$= \dfrac{1{,}99\times10^8}{2\pi(6{,}67\times10^{-11})}.(27800)^3 = \dfrac{4{,}28\times10^{21}}{4{,}19\times10^{-10}} \cong 1{,}02\times10^{31}\,kg$ ✓ // $5.15\,M_\odot$

c) $m_a = 4.15\,m_b$

$m_a + m_b = 1{,}02\times10^{31} \rightarrow m_a = 1{,}02\times10^{31} - m_b$ ∴ $4{,}15\,m_b = 1{,}02\times10^{31} - m_b$

$\rightarrow 4{,}15\,m_b + m_b = 1{,}02\times10^{31} \rightarrow 5{,}15\,m_b = 1{,}02\times10^{31}$ ✓ ∴ $m_b = \dfrac{1{,}02\times10^{31}}{5{,}15} = 1{,}98\times10^{30}\,kg$ ✓ //

E $m_a = 4{,}15\times1{,}98\times10^{30} = 8{,}22\times10^{30}\,Kg$ // $\sim 4.15\,M_\odot$ $\sim 1\,M_\odot$

d) raios individuais

$$\begin{cases} r_s = \frac{v}{2} \cdot (t_b - t_a) \\ r_l = r_s + \frac{v}{2}(t_c - t_b) \end{cases}$$

$v = v_s + v_l = 22,4 + 5,4 = 27,8$ km/s

$r_s = \frac{27,8}{2} \cdot (50112) = 696556,8$ km $\approx 7 \times 10^8$ m $\sim 1 R_\odot$

$\downarrow$ B

$r_l = 696556,8 + \frac{27,8}{2} \cdot (552961) = 696556,8 + 768614,4 = 1465171,2$ km

$\downarrow$ A

$\approx 1,5 \times 10^9$ m

$\sim 2.11 R_\odot$

e) Razão de temperaturas efetivas das duas estrelas T_e

$$\frac{T_B}{T_A} = \left(\frac{F_B}{F_A}\right)^{1/4} \quad e \quad \frac{F_B}{F_A} = \frac{1 - B_P/B_0}{1 - B_S/B_0}$$

Então

$$\frac{B_P}{B_0} = 100^{(m_0 - m_P)/5} = 100^{(5,4 - 9,2)/5} = 100^{-0,76} = 0,03$$

$$\frac{B_S}{B_0} = 100^{(m_0 - m_S)/5} = 100^{(5,4 - 5,44)/5} = 100^{-8e-3} = 0,964$$

Portanto

$$\frac{T_B}{T_A} = \left(\frac{1 - B_P/B_0}{1 - B_S/B_0}\right)^{1/4} = \left(\frac{1 - 0,03}{1 - 0,964}\right)^{1/4} = \left(\frac{0,97}{0,036}\right)^{1/4} = 2,28$$

7) $N_b = N_a$ $\quad$ $n=3$ e $n=1$ ↳ fund $\quad$ ω $E_n = -13,6 \cdot 1/n^2$ eV

$g = 2n^2$

0.8/0.8

a)

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT} \rightarrow 1 = \frac{\cancel{2}\cdot(3)^2}{\cancel{2}(1)^2} e^{-[(-13,6/3^2)-(-13,6/1^2)]/kT}$$

$$\rightarrow 1 = 9\, e^{-[(-1,51)+13,6]/kT} \rightarrow \frac{1}{9} = e^{-12,09/kT} \rightarrow 9^{-1} = e^{-12,09/kT}$$

sumindo com o e

$k = 8,61 \times 10^{-5}$ eV/K

$$\ln(9^{-1}) = -12,09/kT \rightarrow \ln(9) = \frac{12,09}{kT}$$

Então

$$T = \frac{12,09}{8,61\times10^{-5} \cdot \ln(9)} \cong 63907 \text{ K}$$

b) $T = 85400K$ $\therefore$ $kT = 7,35$

$$\frac{N_3}{N_1} = \frac{g_3}{g_1} e^{-(E_3 - E_1)/kT} \rightarrow N_3 = N_1 \cdot \frac{\cancel{2}\cdot(3)^2}{\cancel{2}(1)^2} e^{-12,09/7,35}$$

$$N_3 = N_1 \cdot 9\, e^{-1,65} \cong N_1 \cdot 9 \cdot 0,19 \cong 1,73 N_1$$

8) Ca III / Ca II

Ca II $\rightarrow \chi_{II} = 11{,}9\,eV$

Ca III $\rightarrow Z_{III} = 1$

$T = 5777\,K$

$k = 8{,}61\times10^{-5}\,eV/K$

$P_e = 1{,}5\,N/m^2$

$k = 1{,}38\times10^{-23}\,J/K$

$Z_{II} = 2{,}30$

dados 8.1.5

$h = 6{,}63\times10^{-34}\,J.s$

$h = 4{,}13\times10^{-15}\,eV.s$

$m_e = 9{,}10\times10^{-31}\,kg$

$kT = 0{,}5$ (em eV) e $kT = 1{,}97\times10$

em J/K

0.2 / 0.8

em J/K

$$\left[\frac{N_{III}}{N_{II}}\right]_{Ca} = \frac{2kT\,Z_{III}}{P_e\,Z_{II}}\left(\frac{2\pi\,m_e\,kT}{h^2}\right)^{3/2} e^{-\chi_{II}/kT}$$

em eV/K

$$= \left(\frac{2\cdot kT\cdot 1}{1{,}5\cdot 2{,}30}\right)\left(\frac{2\pi\cdot 9{,}1\times10^{-31}\cdot kT}{(6{,}63\times10^{-34})^2}\right)^{3/2} e^{-11{,}9/0{,}5}$$

?

$$= 4{,}62\times10^{-20}\cdot 1{,}05\times10^{27}\cdot 4{,}6\times10^{-11} = 2{,}24\times10^{-3}$$

UNIDADES?

$$\left[\frac{N_{III}}{N_{II}}\right]_{Ca} = 2{,}03\times10^{-3}$$

Faltou comparar c/ $\left[\frac{N_{II}}{N_{I}}\right]_{Ca} = 918$