AGA0293 Astrofísica Estelar
Profa. Jane Gregorio-Hetem

# Capítulo 16
# Remanescentes degenerados de estrelas

16.1...16.5 Anãs Brancas
16.6 Estrelas de nêutrons
16.7 Pulsares

# Anãs Brancas

- 16.1 A descoberta de Sirius B
- 16.2 Tipos de anãs brancas
- 16.3 A física da matéria degenerada
- 16.4 O limite de Chandrasekhar
- 16.5 O esfriamento das Anãs Brancas

# 16.1 A descoberta de Sirius B

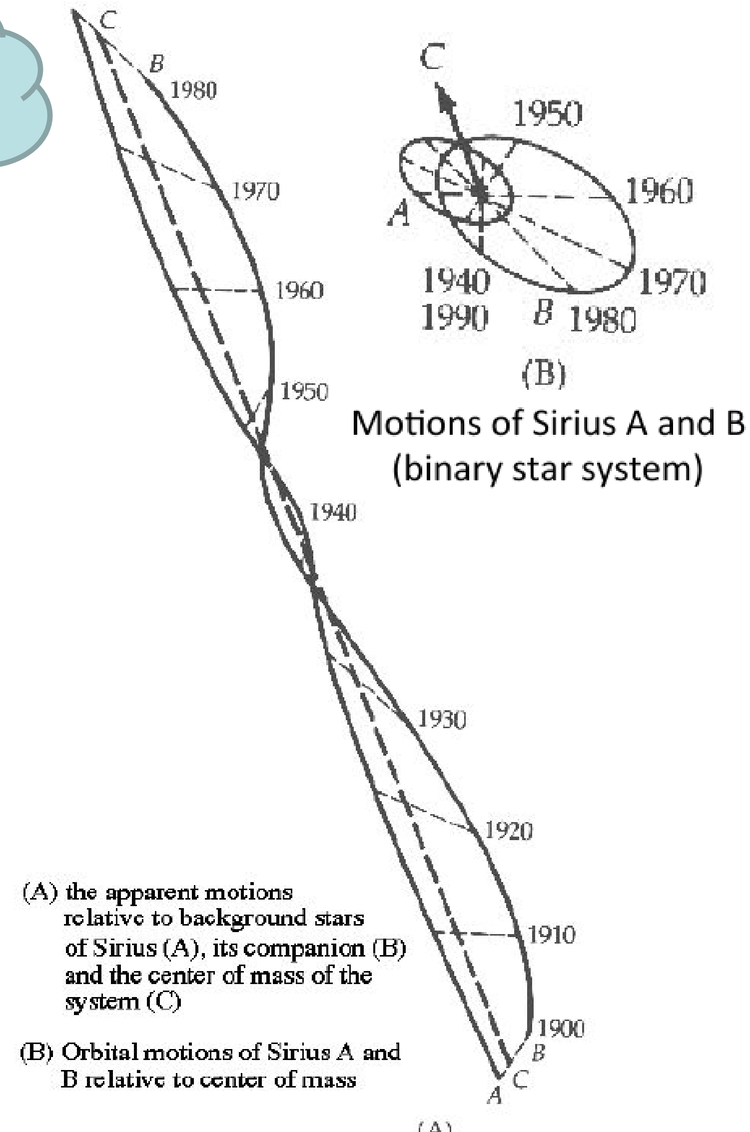
p” = 0,379”
2,64pc

Friedrich Bessel

1838: paralaxe 61 Cyg.

Após 10 anos de pesquisa (1844) → Sirius é binária.

Morreu 16/3/1846 sem conseguir observar Sirius B




Motions of Sirius A and B (binary star system)

Alvan Graham Clark (1832-1897) testou em 31 Jan 1862 um novo refrator de 18,5” (47 cm), o maior da época, e fez a primeira observação de Sirius B

Dearborn Observatory

- Informações detalhadas sobre as órbitas, em torno do centro de massa, permitem determinar as massas individuais.

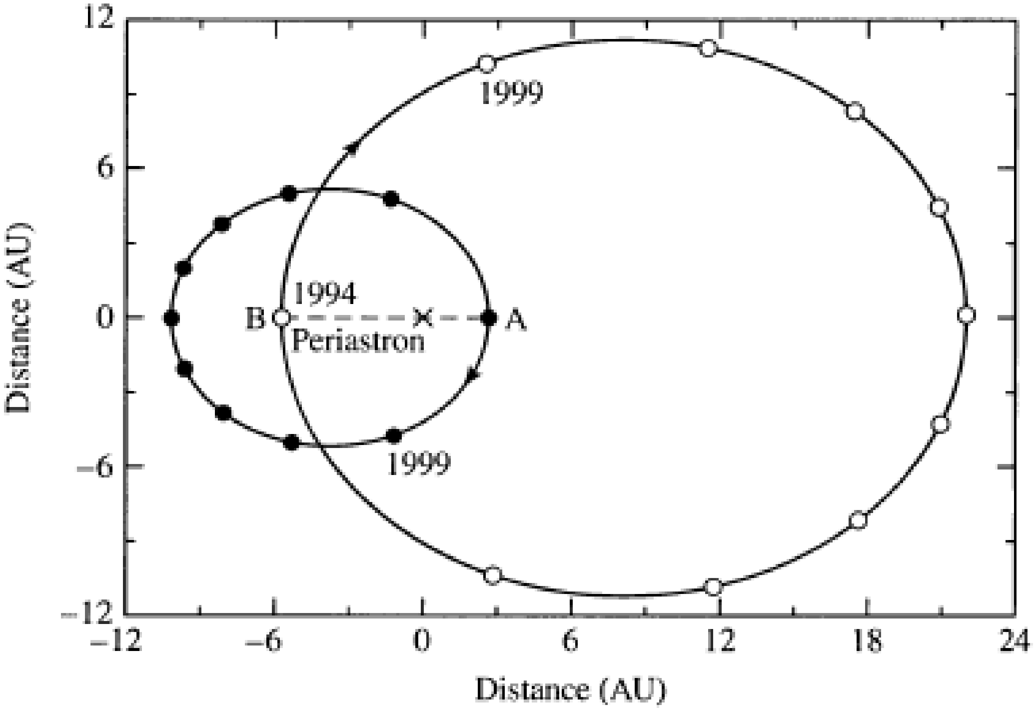
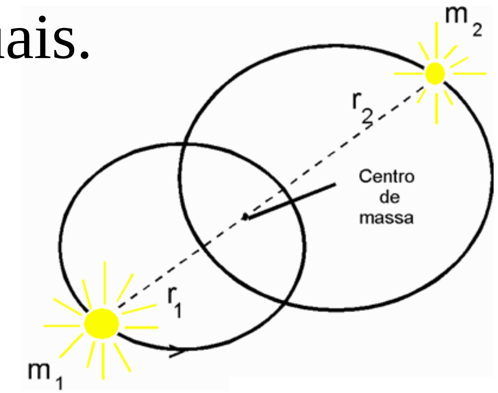



**FIGURE 16.2** The orbits of Sirius A and Sirius B. The center of mass of the system is marked with an "×."




$$a = r_1 + r_2 \qquad \frac{m_1}{m_2} = \frac{r_2}{r_1}$$

$$P^2 = \left[\frac{4\pi^2}{G\left(m_1 + m_2\right)}\right] a^3$$

- Espectroscopia (Adams 1915) revelou $T_B = 27000$ K

$T_A = 9910$ K

$L = 4\,\pi\, R^2\, \sigma\, T^4$

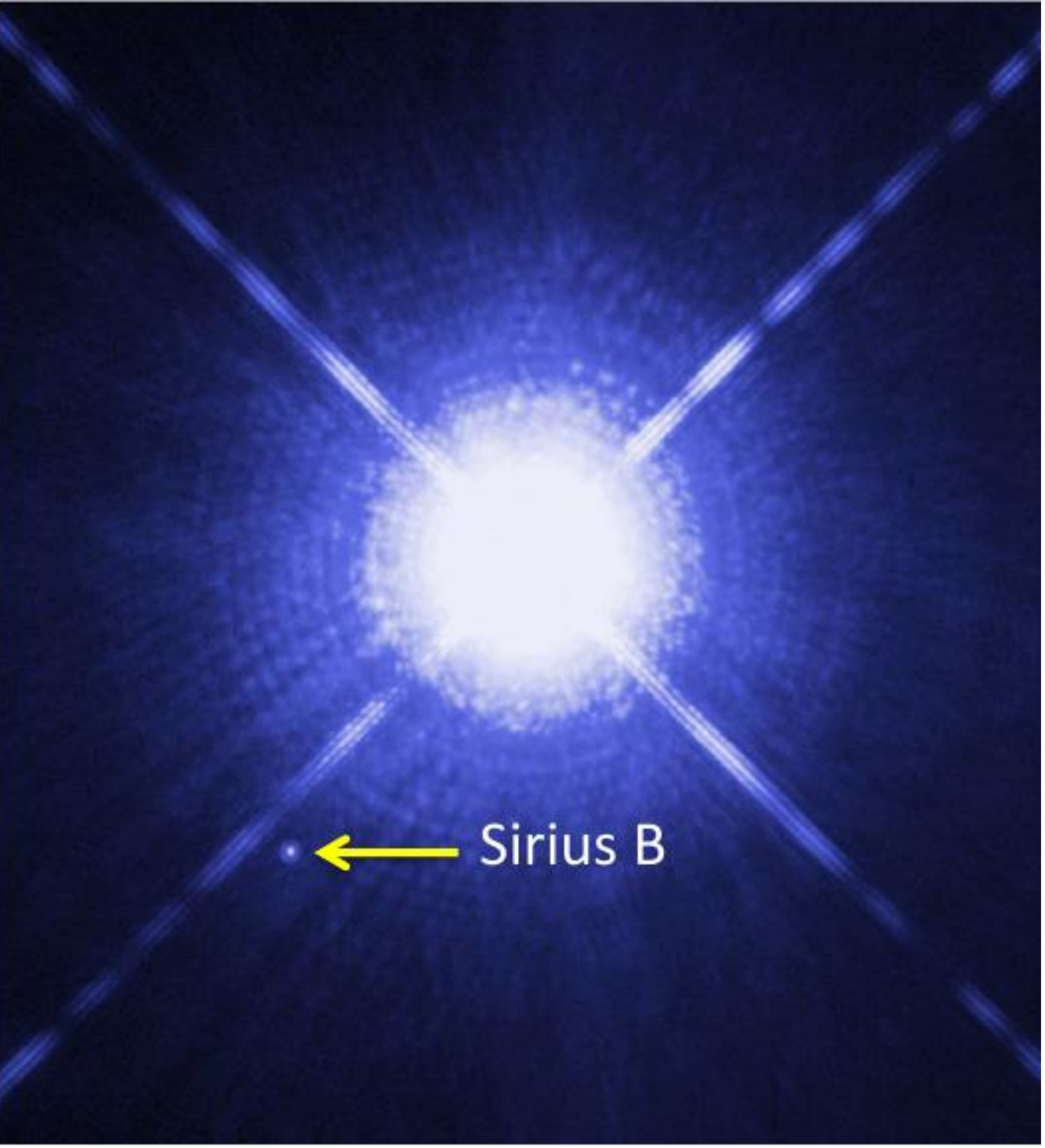
$R_B = 0{,}008\ R_\odot$

$\rho = 3 \times 10^9$ kg m$^{-3}$

# Massas e luminosidades de Sirius A e B

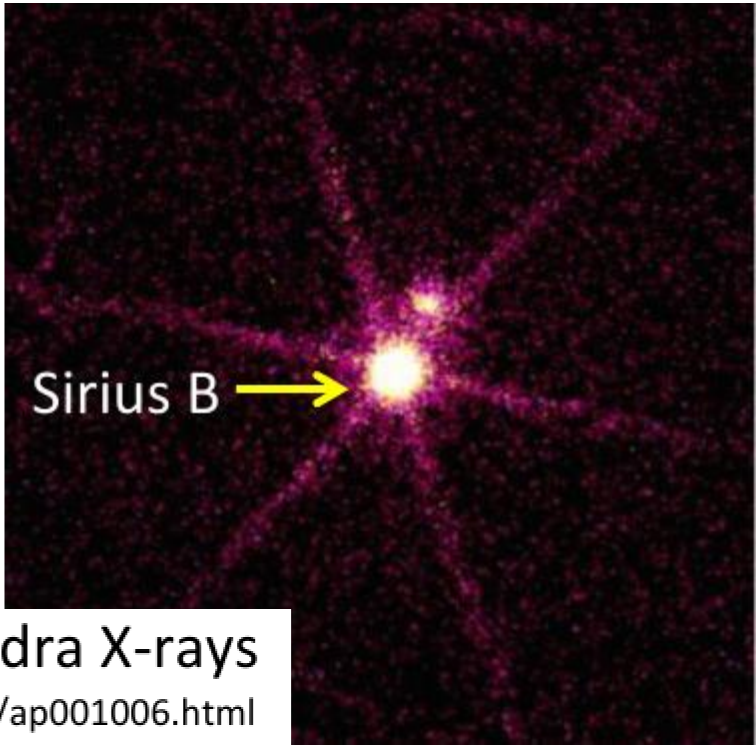
$M_A = 2{,}3\ M_\odot$ ; $L_A = 23{,}5\ L_\odot$

$M_B = 1{,}0\ M_\odot$ ; $L_B = 0{,}03\ L_\odot$




Hubble in the optical

https://www.spacetelescope.org/images/heic0516a/



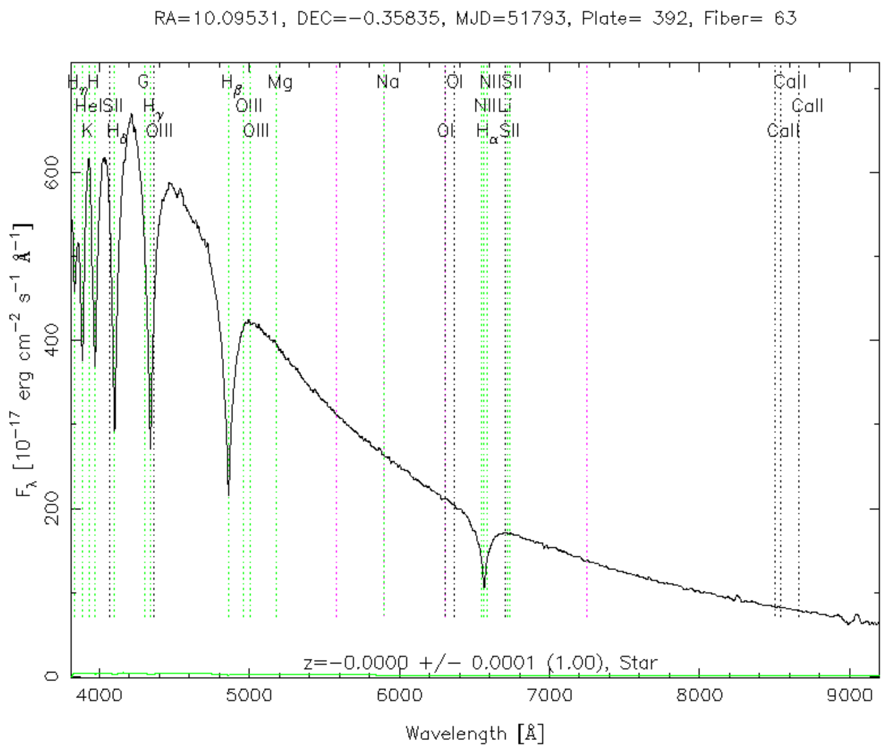

Chandra X-rays

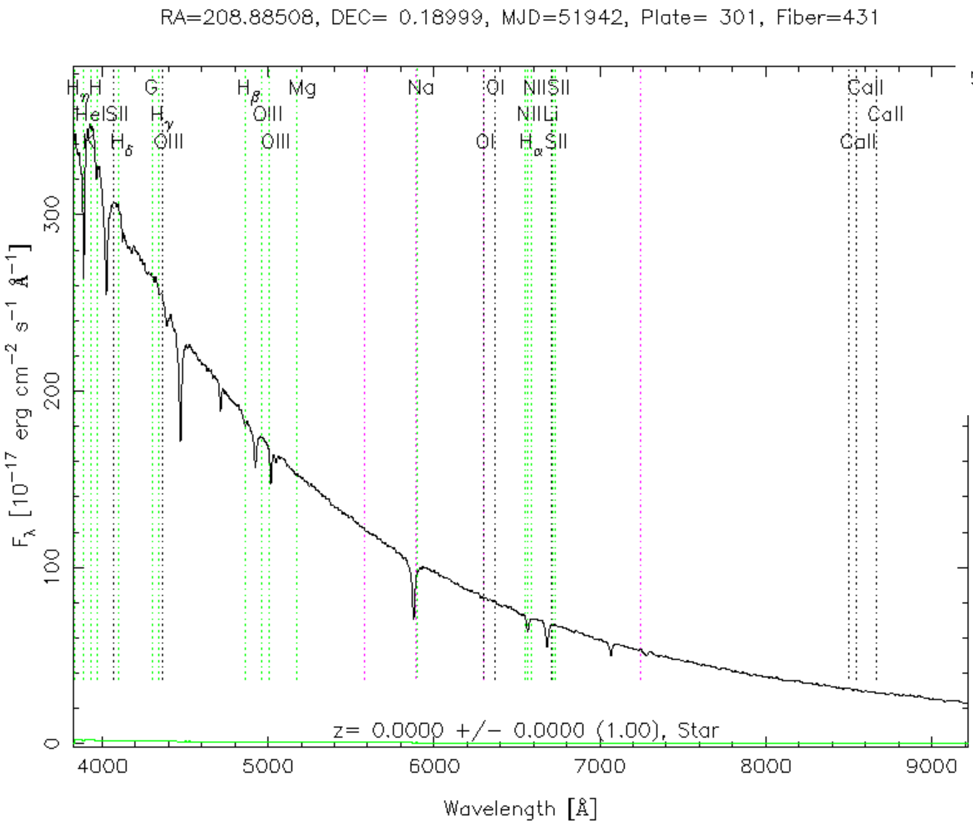
https://apod.nasa.gov/apod/ap001006.html

Alta gravidade ($4.6\ 10^6$ m $s^{-2}$) na superfície da Anã Branca:

espectro com linhas alargadas do H

Anã Branca




Tipo B (Sequência Principal)

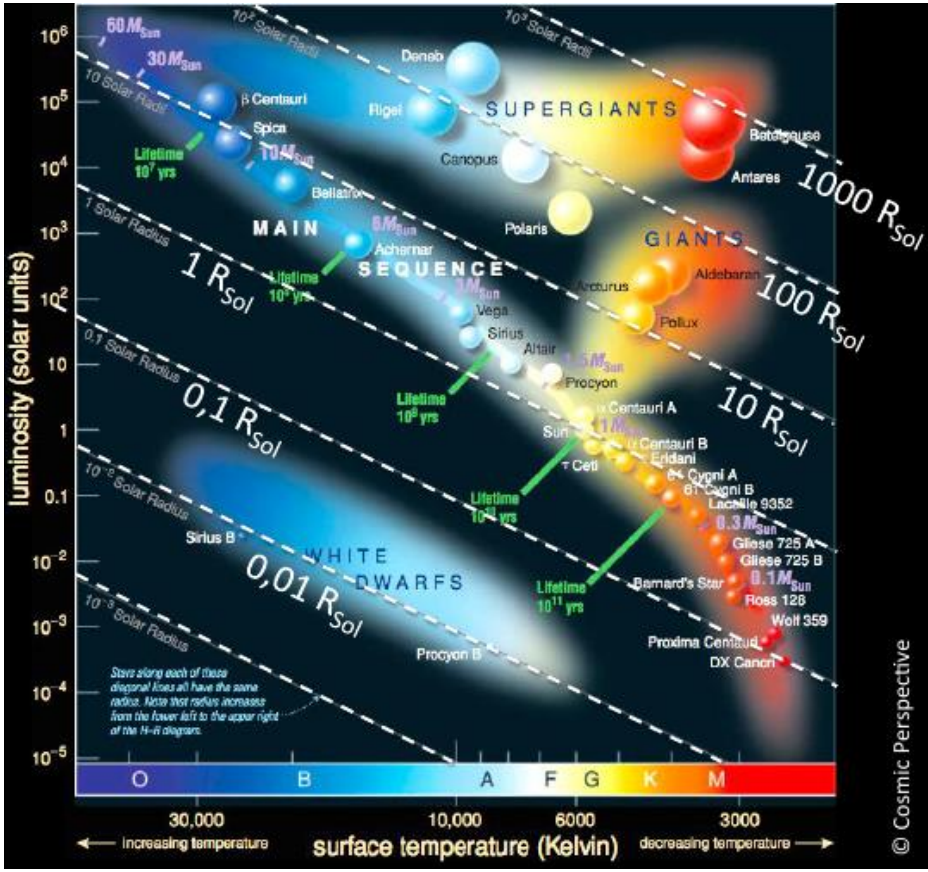
http://classic.sdss.org/dr5/algorithms/spectemplates/

# 16.2 Tipos de Anãs Brancas

T ~5000K a 80000K

diferentes cores

- DA (as mais comuns): linhas do H alargadas.
- DB (8%): sem linhas do H, apenas He.
- DC (14%): sem linhas espectrais.
- DQ (carbono) e DZ (metais)



# Condições físicas no centro das Anãs Brancas

Lembrando a equação do equilíbrio hidrostático:

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{GM(r)}{r^2}\rho = -\dfrac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r^2}\rho \\[2ex] \dfrac{dP}{dr} = -\dfrac{4}{3}\pi G\rho^2 r \end{cases}$$

Integrando e na superfície (r=R) adotando P(R) = 0 e $P_c$=P(0):

$$\begin{cases} P(R) - P(r) = -\dfrac{2}{3}\pi G\rho^2\left(R^2 - r^2\right) \\[2ex] P_c \approx \dfrac{2}{3}\pi G\rho^2 R_{wd}{}^2 \approx 3{,}8\times 10^{22}\, Nm^{-2} \end{cases}$$

# Temperatura central das Anãs Brancas

Usando a expressão do gradiente de temperatura (Eq. 10.68)

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

$$\frac{T_{wd} - T_c}{R_{wd} - 0} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T_c^3}\frac{L_{wd}}{4\pi R_{wd}^2}$$

Supondo que na superfície ($T_{wd} << T_c$) e usando a opacidade média para espalhamento de elétrons (Eq. 9.27): $\bar{\kappa} = 0{,}02 m^2 kg^{-1}$

$$\frac{T_c^4}{R_{wd}} = \frac{3\bar{\kappa}\rho}{4ac}\frac{L_{wd}}{4\pi R_{wd}^2}$$

$$T_c = \left(\frac{3\bar{\kappa}\rho}{4ac}\frac{L_{wd}}{4\pi R_{wd}}\right)^{\frac{1}{4}} \approx 7{,}6 \times 10^7 K$$

*Core* sem H
ou material em fusão

# Massa das Anãs Brancas

Na fase *AGB*, estrelas com *core* de He → $M > 0{,}5\ M_\odot$ → fusão → *core* de C e O ionizados.
Anã Brancas → *core* exposto da estrela original ($M < 8\ M_\odot$).

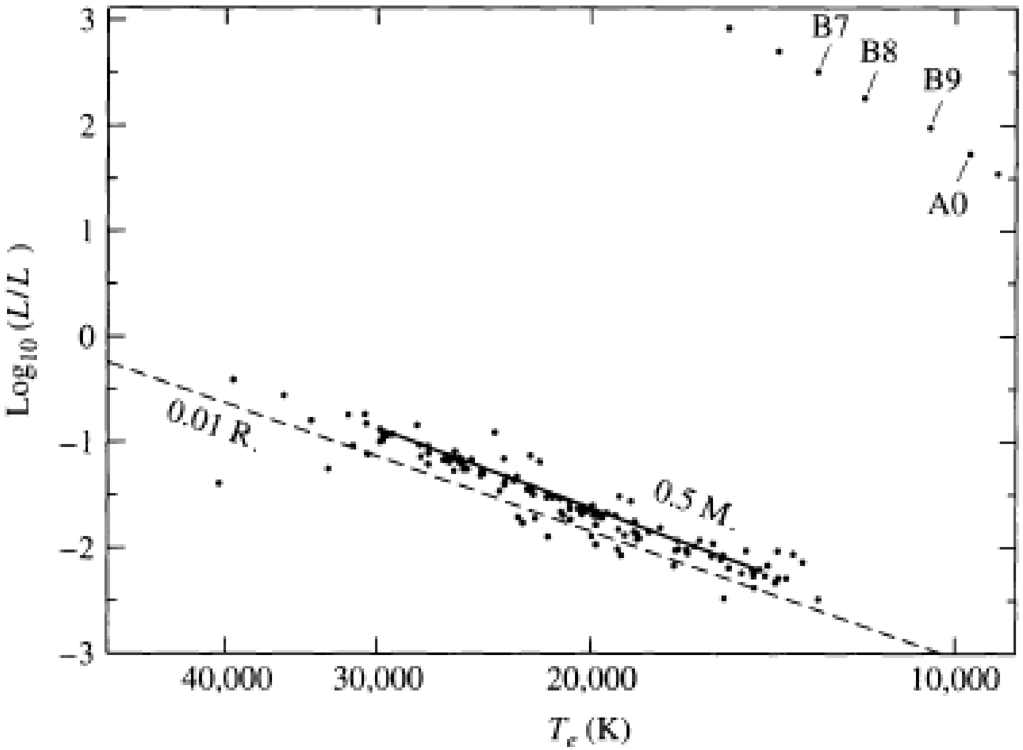



**FIGURE 16.3** DA white dwarfs on an H–R diagram. A line marks the location of the 0.50 $M_\odot$ white dwarfs, and a portion of the main sequence is at the upper right. (Data from Bergeron, Saffer, and Liebert, *Ap. J.*, *394*, 228, 1992.)

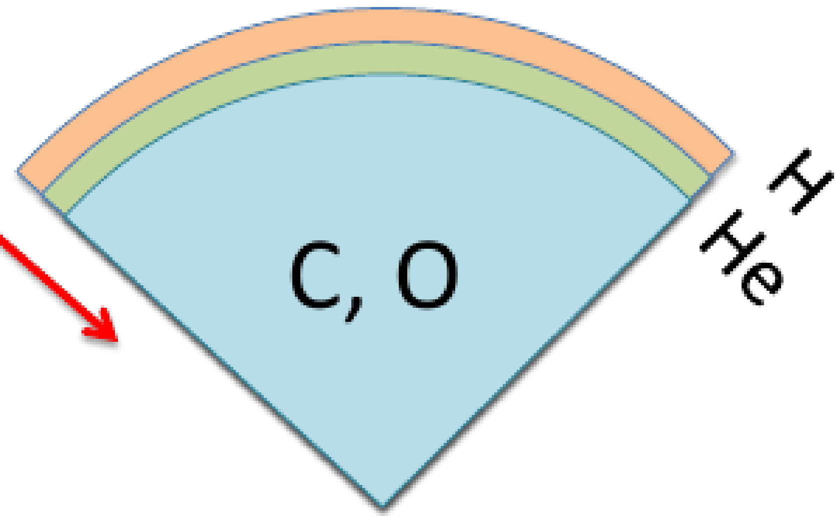
As de tipo DA (80%) têm massas 0,42 a 0,7 $M_\odot$

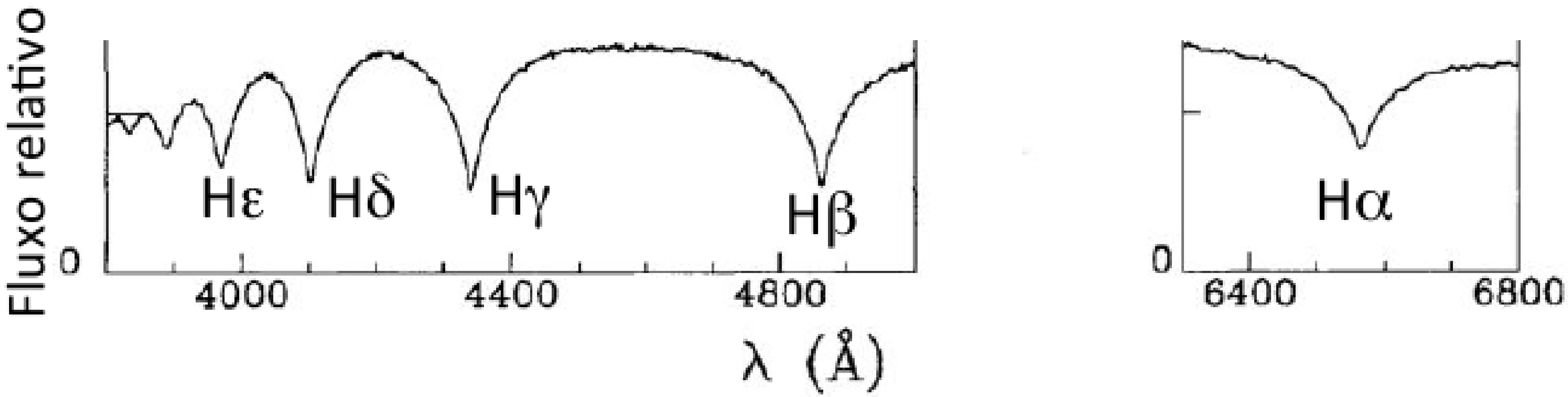
Grandes perdas de massa na fase *AGB* → pulsos térmicos e superventos

# Espectro e composição superficial de Anãs Brancas

Devido à altíssima gravidade → elementos mais pesados para o interior: fina atmosfera do pouco H restante.

Escala de tempo ~100 anos




G 207-9, anã branca de tipo DA4.5

Wesemael et al. 1993 PASP 105, 761

# Anãs Brancas pulsantes

$T_{eff}$ > 12000K → faixa de instabilidade

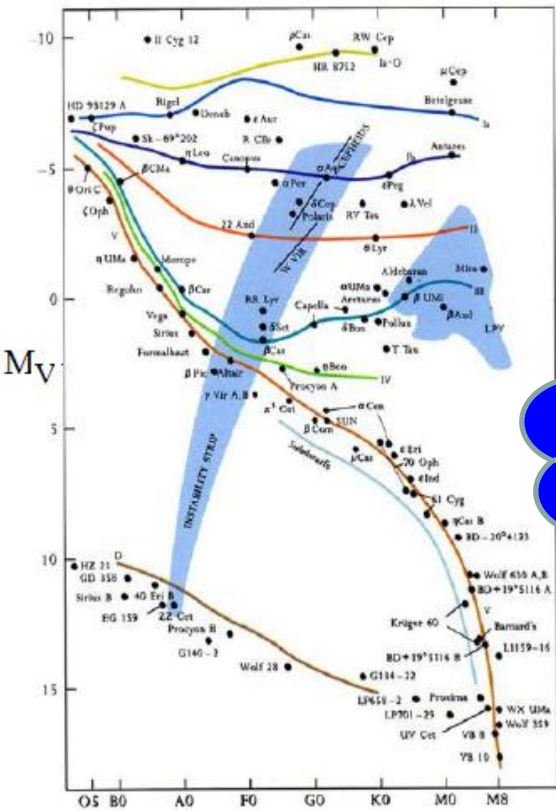



Fig. 8.16

*Núcleo de NP : formação da anã branca*

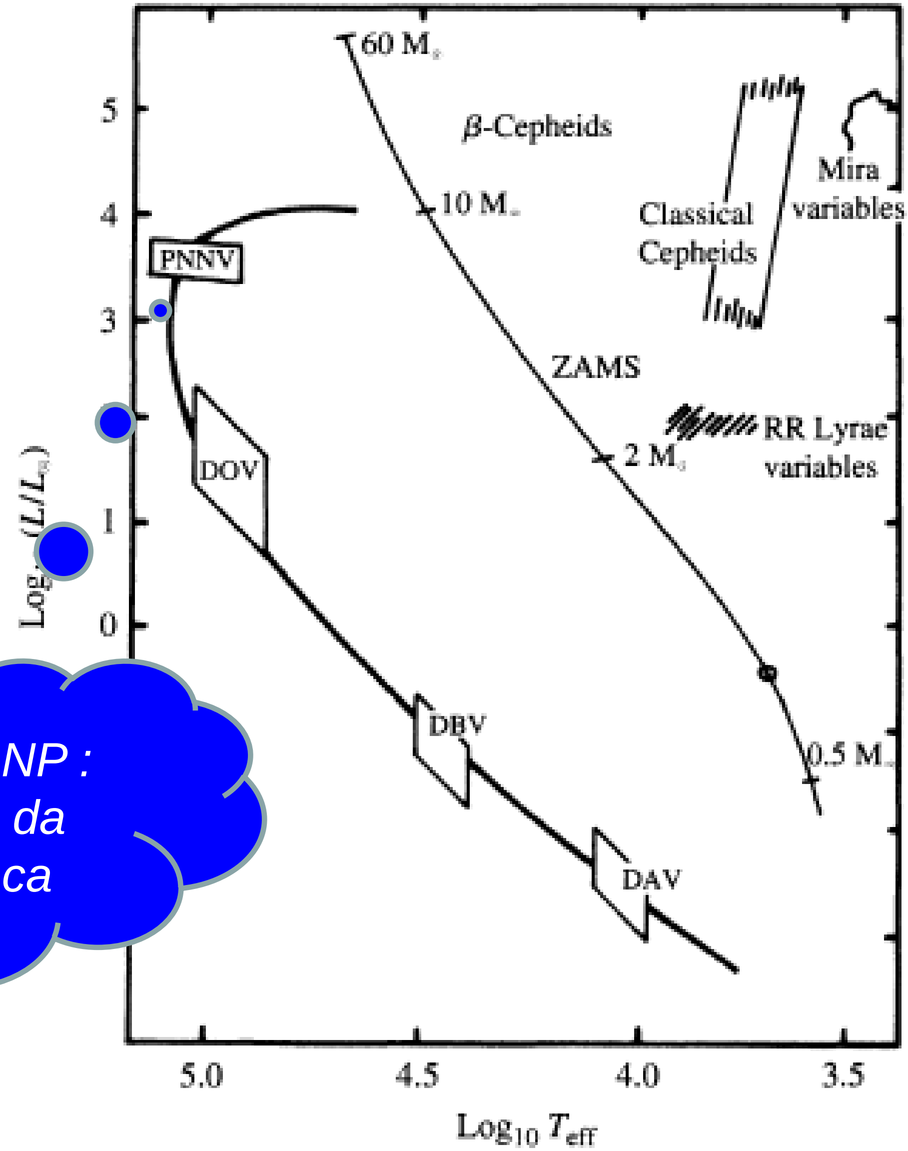



**FIGURE 16.4** Compact pulsators on the H–R diagram. (Figure adapted from Winget, *Advances in Helio- and Asteroseismology*, Christensen-Dalsgaard and Frandsen (eds.), Reidel, Dordrecht, 1988.)

**TABLE 14.1** Pulsating Stars. (Adopted from Cox, *The Theory of Stellar Pulsation*, Princeton University Press, Princeton, NJ, 1980.)

| Type | Range of Periods | Population Type | Radial or Nonradial |
|---|---|---|---|
| Long-Period Variables | 100–700 days | I,II | R |
| Classical Cepheids | 1–50 days | I | R |
| W Virginis stars | 2–45 days | II | R |
| RR Lyrae stars | 1.5–24 hours | II | R |
| $\delta$ Scuti stars | 1–3 hours | I | R,NR |
| $\beta$ Cephei stars | 3–7 hours | I | R,NR |
| ZZ Ceti stars | 100–1000 seconds | I | NR |

Anãs brancas variáveis: DAVs, DBVs...

# 16.3 Física da matéria degenerada

- Gás normal e pressão de radiação não são suficientes para reter o colapso gravitacional nas anãs brancas.
- Princípio de Exclusão de Pauli aplicado aos $e^-$

pressão dos elétrons degenerados

- Matéria completamente degenerada: férmions ocupando todos os estados de menor energia.

Energia de Fermi ($\varepsilon_F$): máxima energia de um $e^-$ num gás completamente degenerado a T = 0K

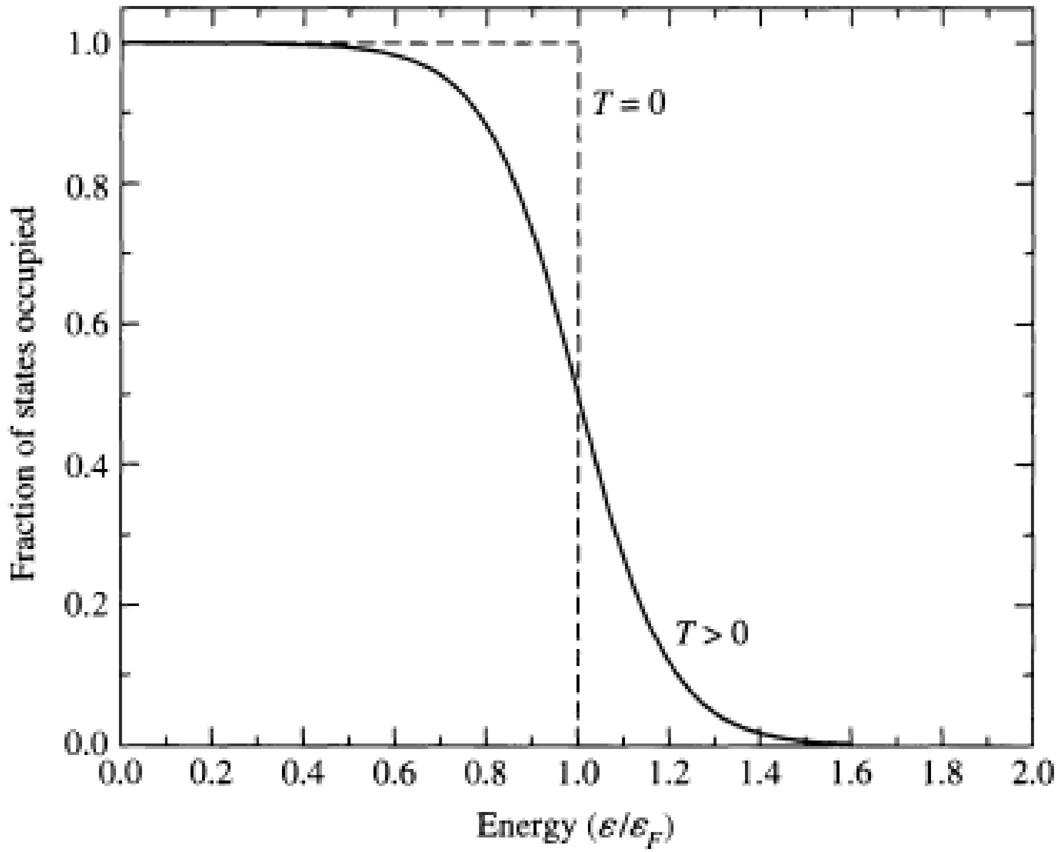



**FIGURE 16.5** Fraction of states of energy $\varepsilon$ occupied by fermions. For $T = 0$, all fermions have $\varepsilon \le \varepsilon_F$, but for $T > 0$, some fermions have energies in excess of the Fermi energy.

Fração de estados de energia ocupados com férmions.
Para T > 0 alguns $e^-$ excedem $\varepsilon_F$

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}}$$

Para deduzir $\varepsilon_F$, suponha $e^-$ distribuídos em uma caixa de lado L. O comprimento de onda em cada dimensão será:

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}}$$

$$\lambda_x = \frac{2L}{N_x}, \quad \lambda_y = \frac{2L}{N_y}, \quad \lambda_z = \frac{2L}{N_z},$$

where $N_x$, $N_y$, and $N_z$ are integer quantum numbers associated with each dimension. Recalling that the de Broglie wavelength is related to momentum (Eq. 5.17),

$$p_x = \frac{hN_x}{2L}, \quad p_y = \frac{hN_y}{2L}, \quad p_y = \frac{hN_x}{2L}.$$

Now, the total kinetic energy of a particle can be written as

$$\varepsilon = \frac{p^2}{2m},$$

where $p^2 = p_x^2 + p_y^2 + p_z^2$. Thus,

$$\varepsilon = \frac{h^2}{8mL^2}(N_x^2 + N_y^2 + N_z^2) = \frac{h^2N^2}{8mL^2}, \qquad (16.2)$$

Lembrando estados de spin: ms= ±½ → $N_T = 2 \times N_{x,y,x}$

spin). Now, each integer coordinate in $N$-space (e.g., $N_x = 1$, $N_y = 3$, $N_z = 1$) corresponds to the quantum state of two electrons. With a large enough sample of electrons, they can be thought of as occupying each integer coordinate out to a radius of $N = \sqrt{N_x^2 + N_y^2 + N_z^2}$, but only for the positive octant of $N$-space where $N_x > 0$, $N_y > 0$, and $N_z > 0$. This means that the total number of electrons will be

$$N_e = 2\left(\frac{1}{8}\right)\left(\frac{4}{3}\pi N^3\right).$$

Solving for $N$ yields

$$N = \left(\frac{3N_e}{\pi}\right)^{1/3}.$$

Substituting into Eq. (16.2) and simplifying, we find that the Fermi energy is given by

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3},} \qquad (16.3)$$

where $m$ is the mass of the electron and $n \equiv N_e/L^3$ is the number of electrons per unit volume. The average energy per electron at zero temperature is $\frac{3}{5}\varepsilon_F$. (Of course the derivation above applies for any fermion, not just electrons.)

# Condição para degenerescência

- Nas altas densidades no interior da anã branca → completa degenerescência para T > 0K (boa aproximação).
- Vamos estimar a energia de Férmi $\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}}$

em função da temperatura e a densidade

Ionização completa: número de elétrons por unidade de volume:

$$n_e = \left(\frac{N_e}{núcleon}\right)\left(\frac{N_{núcleons}}{volume}\right) = \left(\frac{Z}{A}\right)\frac{\rho}{m_H}$$

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{2/3}}$$

# Condição para degenerescência (cont.)

- Comparação com a energia cinética do e⁻:

$$\frac{3}{2}kT < \frac{\hbar^2}{2m_e}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{2/3} \Rightarrow \frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_e k}\left[\frac{3\pi^2}{m_H}\left(\frac{Z}{A}\right)\right]^{2/3}$$

Para Z/A = 0,5
definimos:

$$D = \frac{\hbar^2}{3m_e k}\left[\frac{3\pi^2}{m_H}\left(\frac{Z}{A}\right)\right]^{2/3} = 1261 K m^2 kg^{-2/3}$$

A condição para
degenerescência será:

$$\frac{T}{\rho^{2/3}} < D$$

**Example 16.3.1.** How important is electron degeneracy at the centers of the Sun and Sirius B? At the center of the standard solar model (see Table 11.1), $T_c = 1.570 \times 10^7$ K and $\rho_c = 1.527 \times 10^5$ kg m$^{-3}$. Then

$$\frac{T_c}{\rho_c^{2/3}} = 5500 \text{ K m}^2 \text{ kg}^{-2/3} > \mathcal{D}.$$

In the Sun, electron degeneracy is quite weak and plays a very minor role, supplying only a few tenths of a percent of the central pressure. However, as the Sun continues to evolve, electron degeneracy will become increasingly important (Fig. 16.6). As described in Section 13.2, the Sun will develop a degenerate helium core while on the red giant branch of the H–R diagram, leading eventually to a core helium flash. Later, on the asymptotic giant branch, the progenitor of a carbon–oxygen white dwarf will form in the core to be revealed when the Sun's surface layers are ejected as a planetary nebula.

For Sirius B, the values of the density and central temperature estimated above lead to

$$\frac{T_c}{\rho_c^{2/3}} = 37 \text{ K m}^2 \text{ kg}^{-2/3} \ll \mathcal{D},$$

so complete degeneracy is a valid assumption for Sirius B.

$$\frac{T}{\rho^{2/3}} < D$$

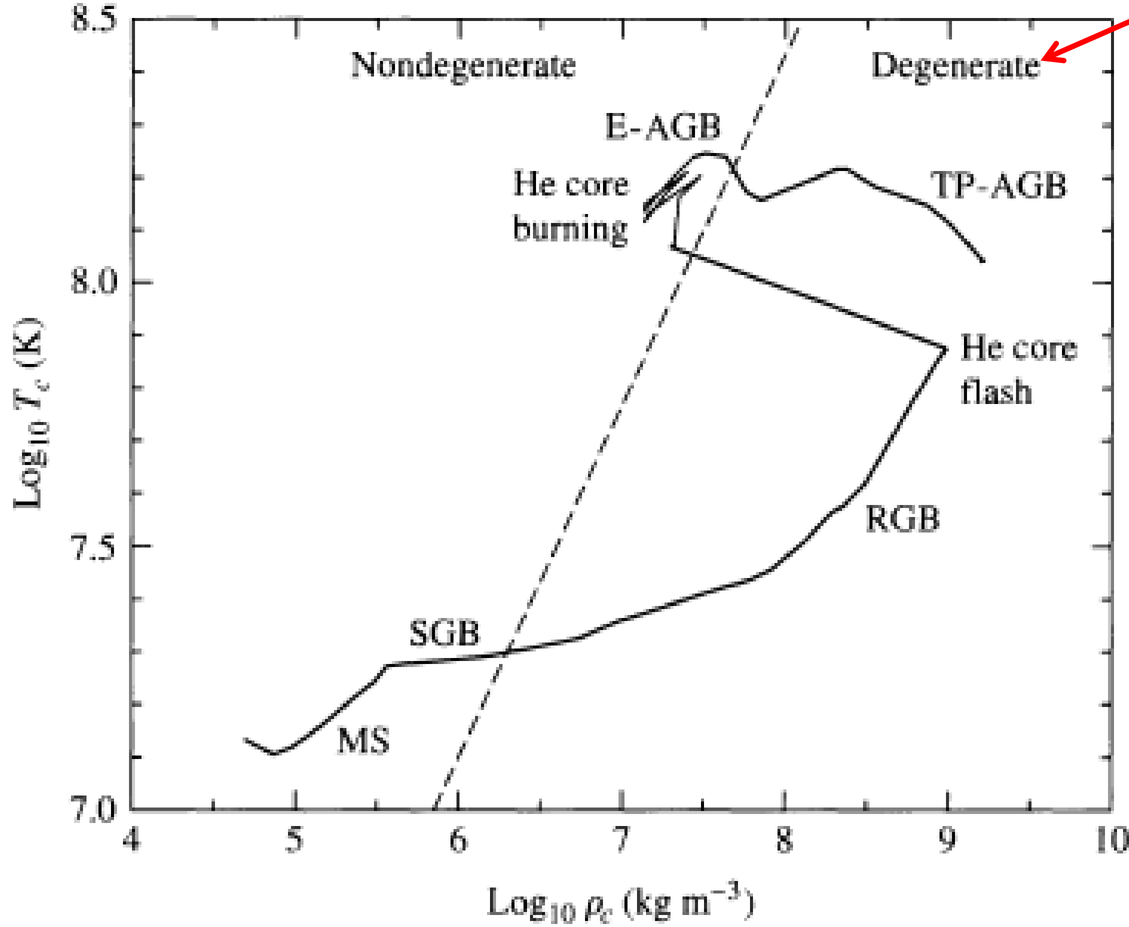



**FIGURE 16.6** Degeneracy in the Sun's center as it evolves. (Data from Mazzitelli and D'Antona, *Ap. J.*, *311*, 762, 1986.)

**Example 16.3.1.** How important is electron degeneracy at the centers of the Sun and Sirius B? At the center of the standard solar model (see Table 11.1), $T_c = 1.570 \times 10^7$ K and $\rho_c = 1.527 \times 10^5$ kg m$^{-3}$. Then

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# Pressão de elétrons degenerados

Devem ser respeitados ambos: princípio de exclusão de Pauli e princípio de incerteza de Heinsenberg:

$$\Delta x \Delta p \approx \hbar$$

A pressão integral (Eq. 10.8) é: $P \approx \dfrac{1}{3} n_e p \mathrm{v}$

Usando

$$\left\{ \begin{aligned} n_e &= \left(\frac{Z}{A}\right)\frac{\rho}{m_H} \\ p_x &\approx \Delta p_x \approx \frac{\hbar}{\Delta x} \approx \hbar n_e^{1/3} \end{aligned} \right.$$

$$P \approx \frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

~2 vezes menor que o valor do que a expressão correta

$$\boxed{P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}}$$

# Pressão de elétrons degenerados (cont.)

$$P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

Para Z/A=0,5 → anã branca tipo C-O (Sirius B)

$P = 1{,}9 \times 10^{22}\ \mathrm{Nm^{-2}}$

→ *pressão de $e^-$ degenerados é responsável por manter a anã branca em equilíbrio.*

# 16.4 Limite de Chandrasekhar: a massa limite das anãs brancas → obtida a partir da relação massa-volume igualando-se a pressão central com a pressão de degenerescência

$$P_c \approx \frac{2}{3}\pi G\rho^2 R_{wd}^2$$

$$P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

$$\frac{2}{3}\pi G\rho^2 R_{wd}^2 = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

$$R_{wd}^2 = \left(\frac{3}{2\pi G\rho^2}\right)\frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

# Limite de Chandrasekhar (cont.)

$$R_{wd}^2 = \left(\frac{3}{2\pi G\rho^2}\right)\frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

$$\rho = \frac{M_{wd}}{4/3\,\pi R_{wd}^3}$$

$$R_{wd}^2 = \left(\frac{3^{1+2/3}}{10}\right)\pi^{4/3-1}\frac{\hbar^2}{Gm_e}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}\rho^{5/3-2}$$

$$R_{wd}^2 = \left(\frac{3^{5/3}}{10}\right)\pi^{1/3}\frac{\hbar^2}{Gm_e}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}\left(\frac{4\pi R_{wd}^3}{3M_{wd}}\right)^{1/3}$$

# Limite de Chandrasekhar (cont.)

$$R_{wd}^2 = \left(\frac{3^{5/3}}{10}\right)\pi^{1/3}\,\frac{\hbar^2}{Gm_e}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}\left(\frac{4\pi R_{wd}^3}{3M_{wd}}\right)^{1/3}$$

$$R_{wd}^2 = \left(\frac{3^{5/3-1/3}}{10}4^{1/3}\right)\pi^{1/3+1/3}\,\frac{\hbar^2}{Gm_e}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}R_{wd}\left(\frac{1}{M_{wd}}\right)^{1/3}$$

$$R_{wd} = \left(\frac{3^{4/3}}{10}2^{2/3}\right)\pi^{2/3}\,\frac{\hbar^2}{Gm_eM_{wd}^{1/3}}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}$$

$$\boxed{R_{wd} = \left(\frac{(18\pi)^{2/3}}{10}\right)\frac{\hbar^2}{Gm_eM_{wd}^{1/3}}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}}$$

# Limite de Chandrasekhar (cont.)

$$\boxed{R_{wd} = \left( \frac{(18\pi)^{2/3}}{10} \right) \frac{\hbar^2}{G m_e M_{wd}^{1/3}} \left[ \left( \frac{Z}{A} \right) \frac{1}{m_H} \right]^{5/3}}$$

M = 1 $M_\odot$
R ~ 2,9× $10^6$m
menor (fator 2) mas compatível com o raio da anã branca.

$$\rho = \frac{M_{wd}}{4/3\, \pi R_{wd}^3} = cte$$

$$M_{wd} R_{wd}^3 = cte$$

$$M_{wd} V_{wd} = cte$$

Relação Massa-Volume

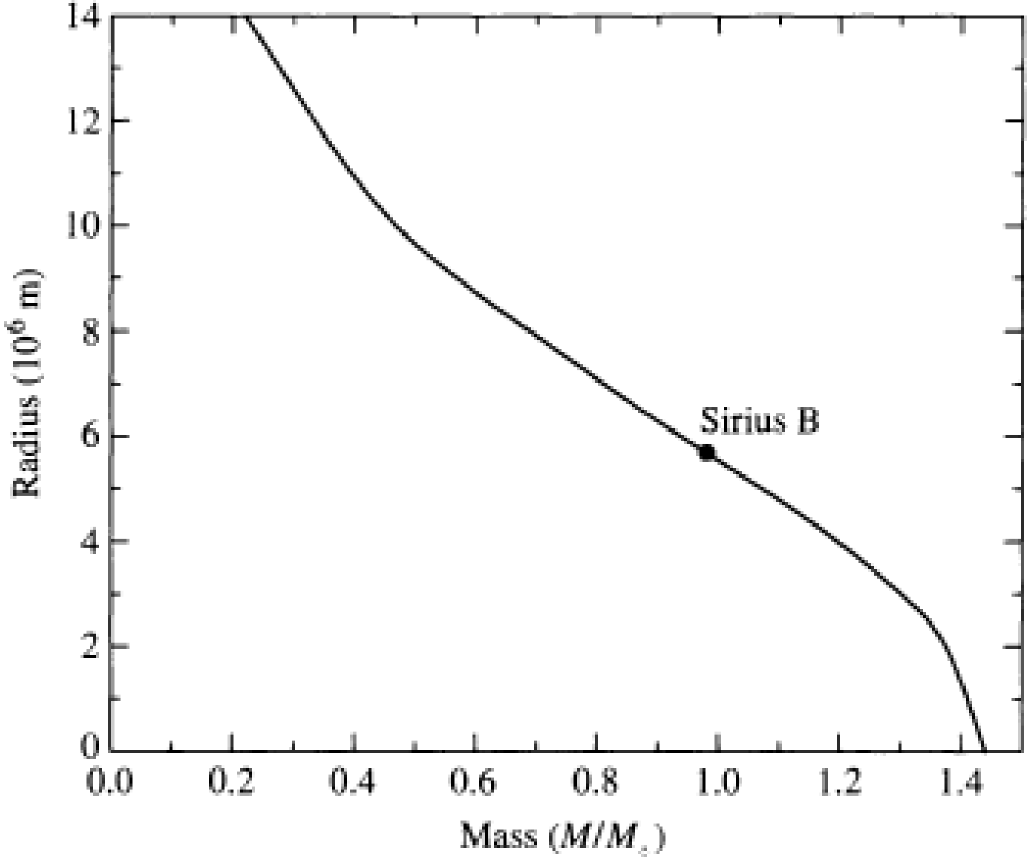



**FIGURE 16.7** Radii of white dwarfs of $M_{wd} \leq M_{Ch}$ at $T = 0$ K.

Massa limite:
$M_{Ch}$=1,44 $M_\odot$

Nenhuma Anã Branca encontrada com M > $M_{Ch}$

# 16.5 O resfriamento das anãs brancas

- Sem fonte de energia (fusão nuclear) no interior da anã branca, a estrela resfria lentamente.
- O transporte de energia mais eficiente é por condução dos elétrons → maior parte da estrela é isotérmica,

circundada por uma camada não-degenerada onde a temperatura cai de forma mais significativa.

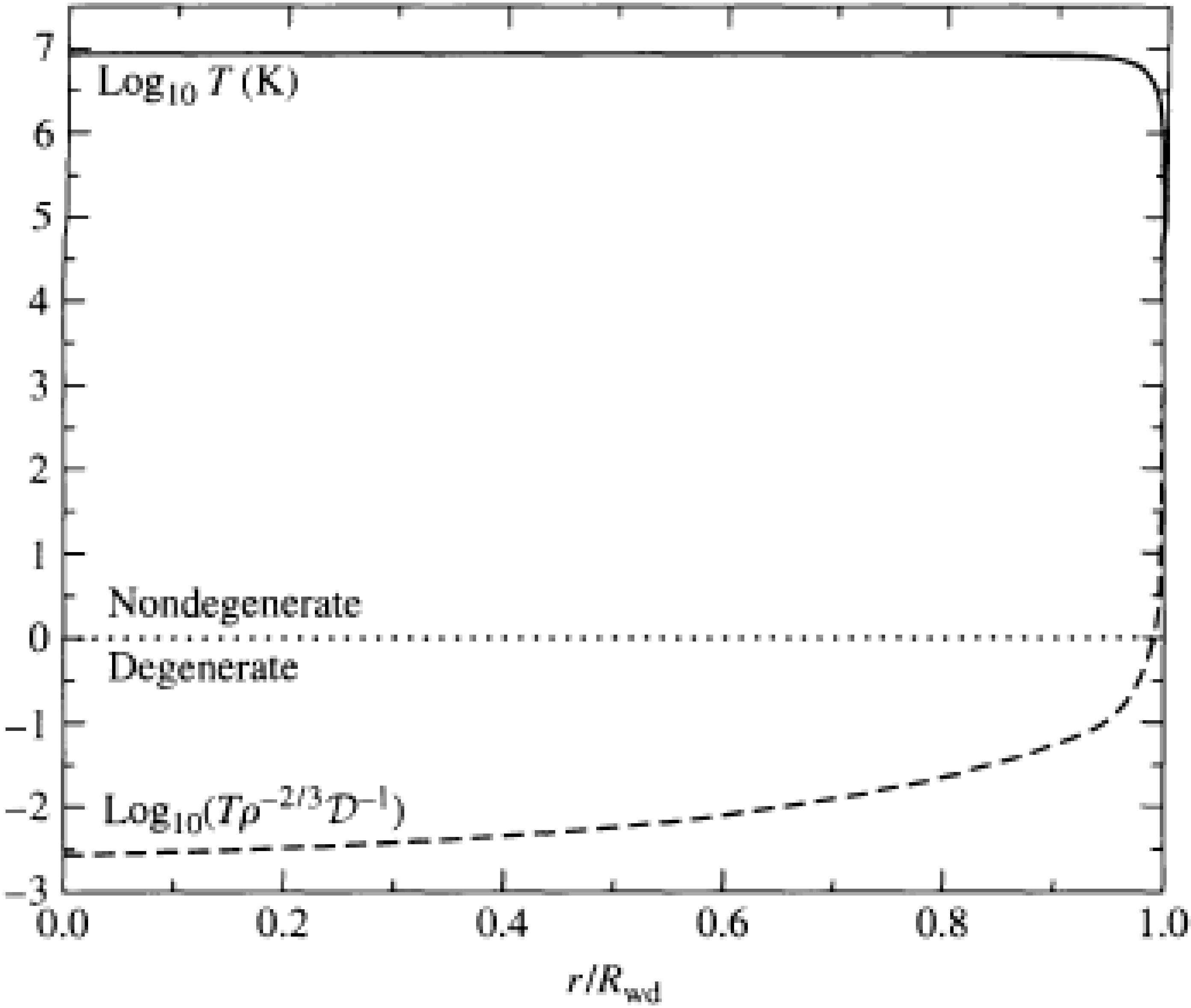



**FIGURE 16.8** Temperature and degree of degeneracy in the interior of a white dwarf model. The horizontal dotted line marks the boundary between degeneracy and nondegeneracy as described by Eq. (16.6).

$$\boxed{\frac{T}{\rho^{2/3}} < D}$$

$$D \equiv 1261 \text{K m}^2 \text{ kg}^{-2/3}$$

## Condições das camadas não degeneradas (Apêndice L)

A pressão do envoltório em função da temperatura:

$$P = \left( \frac{4}{17} \frac{16\pi ac}{3} \frac{GM_{wd}}{L_{wd}} \frac{k}{K_o \mu m_H} \right)^{1/2} T^{17/4}$$

onde coeficiente de opacidade *bound-free* de Kramers (Eq. 9.22) → $K_o = 4{,}34 \times 10^{21}$ Z(1+X) m$^2$ kg$^{-1}$.

Usando a lei do gás ideal (Eq. 10.11):

$$P_g = \frac{\rho k T}{\mu m_H} \Rightarrow \rho = \left( \frac{4}{17} \frac{16\pi ac}{3} \frac{GM_{wd}}{L_{wd}} \frac{\mu m_H}{K_o k} \right)^{1/2} T^{13/4}$$

A transição entre o interior isotérmico ($T_c$) e as camadas superficiais não-degeneradas é dada pela igualdade:

$$\frac{T_c}{\rho^{2/3}} = D \rightarrow T_c = D\rho^{2/3}$$

$$\rho = \left( \frac{4}{17} \frac{16\pi ac}{3} \frac{GM_{wd}}{L_{wd}} \frac{\mu m_H}{K_o k} \right)^{1/2} T^{13/4}$$

$$T_c = D\left( \frac{4}{17} \frac{16\pi ac}{3} GM_{wd} \frac{\mu m_H}{K_o k} \right)^{1/3} \frac{1}{L_{wd}^{1/3}} T_c^{13/6}$$

$$L_{wd}^{1/3} = D\left( \frac{4}{17} \frac{16\pi ac}{3} GM_{wd} \frac{\mu m_H}{K_o k} \right)^{1/3} T_c^{13/6 - 1}$$

$$L_{wd}^{1/3} = D\left(\frac{4}{17}\frac{16\pi ac}{3}GM_{wd}\frac{\mu m_H}{K_o k}\right)^{1/3} T_c^{13/6-1}$$

$$L_{wd} = D^3\left(\frac{4}{17}\frac{16\pi ac}{3}GM_{wd}\frac{\mu m_H}{K_o k}\right)\left(T_c^{7/6}\right)^3$$

$$L_{wd} = \frac{4D^3}{17}\frac{16\pi ac}{3}GM_{wd}\frac{\mu m_H}{K_o k}T_c^{7/2} \Rightarrow \boxed{L_{wd} = CT_c^{7/2}}$$

$$C \equiv \frac{4D^3}{17}\frac{16\pi ac}{3}G\frac{\mu m_H}{K_o k}M_{wd} \Rightarrow C \equiv 6{,}65\times10^{-3}\frac{M_{wd}}{M_{sun}}\frac{\mu}{Z(1+X)}$$

**Example 16.5.1.** Equation (16.19) can be used to estimate the interior temperature of a 1 $M_\odot$ white dwarf with $L_{wd} = 0.03\ L_\odot$. Arbitrarily assuming values of $X = 0$, $Y = 0.9$, $Z = 0.1$ for the nondegenerate envelope (so $\mu \simeq 1.4$) results in[16]

$$T_c = \left[\frac{L_{wd}}{6.65 \times 10^{-3}} \left(\frac{M_\odot}{M_{wd}}\right) \frac{Z(1+X)}{\mu}\right]^{2/7} = 2.8 \times 10^7\ \mathrm{K}.$$

Equating the two sides of the degeneracy condition, Eq. (16.6), shows that the density at the base of the nondegenerate envelope is about

$$\rho = \left(\frac{T_c}{\mathcal{D}}\right)^{3/2} = 3.4 \times 10^6\ \mathrm{kg\ m^{-3}}.$$

This result is several orders of magnitude less than the average density of a 1 $M_\odot$ white dwarf such as Sirius B and confirms that the envelope is indeed thin, contributing very little to the star's total mass.

# Escala de tempo de esfriamento

A energia térmica da anã branca é dada pela energia cinética de seus núcleos.

Supondo composição uniforme, o número de núcleos é a massa da estrela ($M_{wd}$) dividida pela massa de 1 núcleo ($Am_H$), e a energia disponível para radiação será:

$$U = \frac{M_{wd}}{Am_H} \frac{3}{2} kT_c$$

Para $T_c$= 2,8 $\times 10^7$K
e A=12 (carbono),
U= 6 $\times$ $10^{40}$ J

Para estimar o tempo de esfriamento dividimos a energia pela luminosidade da anã branca:

$$\boxed{L_{wd} = CT_c^{7/2}}$$

$$\tau_{cool} = \frac{U}{L_{wd}} = \frac{3}{2} \frac{M_{wd} k}{Am_H CT_c^{5/2}}$$

## Escala de tempo de esfriamento (cont.)

$$\tau_{cool} = \frac{3}{2}\frac{M_{wd}k}{Am_H C T_c^{5/2}}$$

$$\tau_{cool} = 5{,}2\times10^{15}\,s \approx 1{,}7\times10^{8}\,anos$$

resultado sub-estimado, já que $\tau_{cool}$ aumenta conforme $T_c$ diminui.

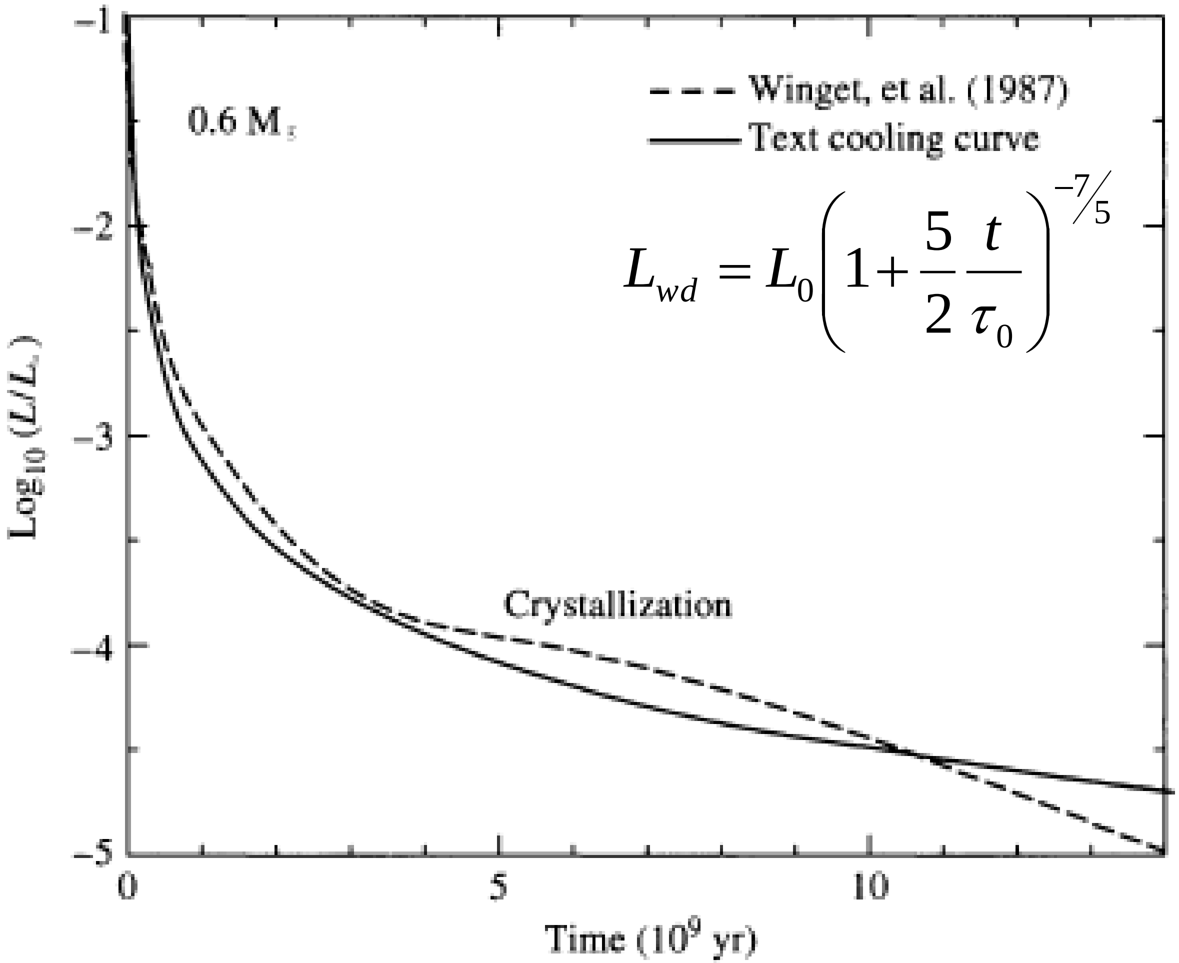
## Variação da Luminosidade com o tempo:

$$-\frac{dU}{dt} = L_{wd}$$

$$L_{wd} = L_0\left(1+\frac{5}{2}\frac{t}{\tau_0}\right)^{-7/5}$$

onde o valor inicial é

$$L_0 = CT_0^{7/2}$$




$$L_{wd} = L_0 \left( 1 + \frac{5}{2} \frac{t}{\tau_0} \right)^{-7/5}$$

**FIGURE 16.9** Theoretical cooling curves for 0.6 $M_\odot$ white-dwarf models. [The solid line is from Eq. (16.23), and the dashed line is from Winget et al., *Ap. J. Lett.*, *315*, L77, 1987.]

O resfriamento da estrela é seguido da cristalização do carbono e do oxigênio ("estrela de diamante").

# Próxima aula

- 16.6 Estrelas de nêutrons
- 16.7 Pulsares