EUF

Exame Unificado

das Pós-graduações em Física

Para o primeiro semestre de 2019

Gabarito

Q1.

a) Conservação do momento linear:

$$\vec{p}_i = \vec{p}_f \Rightarrow p_{ix} = p_{fx} \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$0 + m(-v_0) = 2m v_{1f} + m v_{2f}$$

$$v_{2f} = -v_0 - 2v_{1f} \qquad (1)$$

Conservação da energia:

$$E_{ci} = E_{cf} \Rightarrow \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$
$$0 + \frac{1}{2} m v_0^2 = \frac{1}{2} 2m v_{1f}^2 + \frac{1}{2} m v_{2f}^2$$

$$v_0^2 = 2v_{1f}^2 + v_{2f}^2 \qquad (2)$$

(1) em (2):

$$v_0^2 = 2v_{1f}^2 + \left(v_0 + 2v_{1f}\right)^2 = 2v_{1f}^2 + v_0^2 + 4v_0 v_{1f} + 4v_{1f}^2$$
$$6v_{1f}^2 + 4v_0 v_{1f} = 0$$
$$v_{1f}(3v_{1f} + 2v_0) = 0$$

$v_{1f} = 0$ (o bloco 2 passa pelo bloco 1 sem colidir)

ou

$$v_{1f} = -\frac{2}{3} v_0 \qquad (3)$$

(3) em (1):

$$v_{2f} = -v_0 - 2v_{1f} = -v_0 - 2\left(-\frac{2}{3} v_0\right) = \frac{1}{3} v_0$$

Os vetores ficam: $\vec{v}_{1f} = -\frac{2}{3} v_0\, \hat{x}$ e $\vec{v}_{2f} = \frac{1}{3} v_0\, \hat{x}$

b) Conservação da energia:

$$\frac{1}{2} k\, x_m^2 = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} 2m \left(-\frac{2}{3} v_0\right)^2 \Rightarrow x_m = \sqrt{\frac{2m}{k}}\, \frac{2}{3} v_0$$

c) Teorema do trabalho-energia cinética:

$$W_{peso} + W_{atrito} = E_{cf} - E_i = 0 - E_i$$

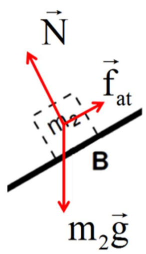
$$-m_2 gh - \mu Nd = -\frac{1}{2} m_2 v_{2f}^2$$

$$N = mg\cos\theta$$

$$mgh + \mu mgd\cos\theta = \frac{1}{2} m\left(\frac{1}{3}v_0\right)^2 \Rightarrow \mu = \frac{\left(\frac{v_0^2}{18} - gh\right)}{gd\cos\theta} = \frac{h}{d\cos\theta}\left(\frac{v_0^2}{18gh} - 1\right)$$

$$\boxed{\mu = tg\theta\left(\frac{v_0^2}{18gh} - 1\right)}$$

d)



Q2.

a) Energia potencial:

$$V(\vec{r}) = -\int_{ref}^{\vec{r}} F(\vec{r}\,')\cdot d\vec{\ell}\,' = -\int_{ref}^{\vec{r}} m\gamma\hat{\mathrm{x}}\cdot d\vec{\ell}\,' = -m\gamma\int_{ref}^{\vec{r}} \hat{\mathrm{x}}\cdot d\vec{\ell}\,'$$

$V(0) = 0$:

$$\boxed{V(\vec{r}) = -m\gamma\int_0^x dx' = -m\gamma x}$$

b) Equação do vínculo: $\boxed{\theta(t) = \omega t + \phi, \quad \phi = cte}$.

A força de vínculo é a força aplicada pela barra.

Note que a força é normal à barra, visto que não há atrito.

c) Energia cinética: $T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\omega^2$

Energia potencial: $V = -m\gamma x = -m\gamma r\cos\theta = -m\gamma r\cos(\omega t + \phi)$

Lagrangiana: $\boxed{L = T - V = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\omega^2 + \gamma mr\cos(\omega t + \phi)}$

Equação de movimento: $\frac{\partial L}{\partial \dot{r}} = m\dot{r}; \qquad \frac{\partial L}{\partial r} = mr\omega^2 + \gamma m\cos(\omega t + \phi)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \left(\frac{\partial L}{\partial r}\right) = 0 \qquad \Rightarrow \quad m\ddot{r} - mr\omega^2 - \gamma m\cos(\omega t + \phi) = 0$$

$$\boxed{\ddot{r} - \omega^2 r = \gamma\cos(\omega t + \phi)}$$

d) Solução da equação de movimento:

$$\gamma = 0 \quad \Rightarrow \qquad \ddot{r} - \omega^2 r = 0$$

$$\lambda^2 - \omega^2 = 0 \Rightarrow \lambda = \pm\omega$$

$$r(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} = Ae^{\omega t} + Be^{-\omega t}$$

$$r(0) = a \Rightarrow A + B = a$$

$$\dot{r}(0) = 0 \Rightarrow \omega(A - B) = 0 \Rightarrow A = B = a/2$$

$$\boxed{r(t) = a\left(\frac{e^{\omega t} + e^{-\omega t}}{2}\right) = a\cosh(\omega t)}$$

Note que $\theta(0) = \phi = 0 \quad \Rightarrow \qquad \theta(t) = \omega t$

Q3.

a) Equação de Schrödinger em duas dimensões:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,y)+V(x,y)\Psi(x,y)=E\Psi(x,y)$$

$V=0$ dentro da caixa.

$$\boxed{-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi(x,y)}{\partial x^2}+\frac{\partial^2\Psi(x,y)}{\partial y^2}\right)=E\Psi(x,y)}$$

b)

$$\Psi(x,y)=\psi(x)\phi(y)$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi}\frac{\partial^2\psi(x)}{\partial x^2}+\frac{1}{\phi}\frac{\partial^2\phi(y)}{\partial y^2}\right)=E$$

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2}=-k_x^2 \quad \Rightarrow \psi(x)=A\operatorname{sen}(k_x x)$$

$$\psi(0)=\psi(a)=0\Rightarrow k_x=\frac{n_x\pi}{a},\quad n_x=1,2...$$

$$\frac{1}{\phi}\frac{\partial^2\phi}{\partial y^2}=-k_y^2\Rightarrow \phi(y)=B\operatorname{sen}(k_y y)$$

$$\phi(0)=\phi(b)=0\Rightarrow k_y=\frac{n_y\pi}{b},\quad n_y=1,2...$$

$$\Psi(x,y)=AB\operatorname{sen}\left(\frac{n_x\pi x}{a}\right)\operatorname{sen}\left(\frac{n_y\pi y}{b}\right)$$

Normalização:

$$\int_0^a dx\int_0^b dy\ \Psi^*(x,y)\ \Psi(x,y)=1$$

$$(AB)^2\int_0^a dx\operatorname{sen}^2\left(\frac{n_x\pi x}{a}\right)\int_0^b dy\operatorname{sen}^2\left(\frac{n_y\pi y}{b}\right)=1$$

$$(AB)^2\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)=1\Rightarrow AB=\frac{2}{\sqrt{ab}}$$

$$\boxed{\Psi_{n_x n_y}(x,y)=\frac{2}{\sqrt{ab}}\operatorname{sen}\left(\frac{n_x\pi x}{a}\right)\operatorname{sen}\left(\frac{n_y\pi y}{b}\right),\quad n_x,n_y=1,2...}$$

$$k_x^2+k_y^2=\frac{2mE}{\hbar^2}\Rightarrow E=\frac{\hbar^2}{2m}\left(k_x^2+k_y^2\right)$$

$$E_{n_x n_y} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

c)

$$E_{11} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

$$\text{Probabilidade}_{11} = \frac{C^2}{C^2 + D^2}$$

Se $\Phi(x, y)$ estiver normalizada, então $C^2 + D^2 = 1$, e

$$\text{Probabilidade}_{11} = C^2$$

$$E_{12} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{1}{a^2} + \frac{4}{b^2}\right)$$

$$\text{Probabilidade}_{12} = D^2$$

d) Não, pois é um estado de superposição de auto-estados de energias diferentes.

$$\Phi(x, y, t) = e^{-i\frac{\hat{H}}{\hbar}t}\Phi(x, y, t = 0) = Ce^{-i\frac{E_{11}}{\hbar}t}\Psi_{11}(x, y) + De^{-i\frac{E_{12}}{\hbar}t}\Psi_{12}(x, y)$$

Q4.

a) Lei de Wien:

$$\lambda_{\max_1} T_1 = W = 2{,}898\times10^{-3}\ \mathrm{m\cdot K}$$

Do gráfico,

$$\lambda_{\max_1} = 3000\,\mathrm{nm} = 3{,}0\times10^{-6}\ \mathrm{m}$$

$$T_1 = \frac{2{,}898\times10^{-3}\ \mathrm{m\cdot K}}{3{,}0\times10^{-6}\ \mathrm{m}} = 966\,\mathrm{K}$$

b)

$$R_T = \sigma T^4$$

$$R_T = 5{,}67\times10^{-8}\frac{\mathrm{W}}{\mathrm{m^2K^4}}\times(966\,\mathrm{K})^4 = 4{,}9\times10^4\,\frac{\mathrm{W}}{\mathrm{m^2}}$$

c)

$$R_{6000-8000} = \text{área grf}_{6000-8000}\ \frac{4{,}9\times10^4\,\frac{\mathrm{W}}{\mathrm{m^2}}}{\text{área grf}_{\text{total}}}$$

$$\text{área grf}_{6000-8000} = 4\,\square$$

$$\text{área grf}_{\text{total}} = 31\,\square$$

$$R_{6000-8000} = \frac{4}{31}\times4{,}9\times10^4\,\frac{\mathrm{W}}{\mathrm{m^2}} = 6{,}3\times10^3\,\frac{\mathrm{W}}{\mathrm{m^2}}$$

d)

$$T_2 = 3T_1$$

A Lei de Wien resulta em:

$$\lambda_{\max_1} T_1 = \lambda_{\max 2} T_2 \Rightarrow \lambda_{\max 2} = \frac{T_1}{T_2}\lambda_{\max_1} = \frac{1}{3}\lambda_{\max_1}$$

$$\lambda_{\max 2} = \frac{3000\,\mathrm{nm}}{3} = 1000\,\mathrm{nm}$$

Q5.

a) Expansão livre: $Q = 0, W = 0 \Rightarrow \Delta U = 0$

Gás ideal: $U = U(T) \Rightarrow \Delta T = 0$

$T = cte \Rightarrow P_0 V_0 = P_1 V_1 = P_1 (5V_0)$

$$\boxed{P_1 = P_0/5}$$

b) Processo reversível ligando (0) a (1): expansão isotérmica

$dS = \frac{\bar{d}Q}{T} = \frac{PdV}{T}$, pois $dU = 0$.

$\frac{P}{T} = \frac{nR}{V} \Rightarrow dS = nR\frac{dV}{V} \Rightarrow \Delta S_{0\to 1} = nR\ln\left(\frac{V_1}{V_0}\right)$

$$\boxed{\Delta S_{0\to 1} = (2\,\text{mols})\,R\ln(5) = 16,6\ln(5)\frac{\text{J}}{\text{K}}}$$

c)

$P_1 V_1^{\gamma} = P_f V_f^{\gamma}$

$\frac{P_0}{5}(5V_0)^{\gamma} = 5^{2/5} P_0 (V_0)^{\gamma}$

$5^{\gamma-1} = 5^{2/5} \Rightarrow \gamma - 1 = 2/5$

$\boxed{\gamma = 7/5 = 1,4}$. O gás é diatômico.

d)

$U = C_V T = nc_V T \quad \Rightarrow \quad \frac{U_f}{U_i} = \frac{T_f}{T_i}$

$\frac{U_f}{U_i} = \frac{P_f V_f}{P_0 V_0} = \frac{5^{2/5} P_0 V_0}{P_0 V_0}$

$$\boxed{\frac{U_f}{U_i} = 5^{2/5} = 1,9}$$

Q6.

a)

$$I = \frac{|\varepsilon|}{R}; \quad |\varepsilon| = \left|\frac{d\Phi_B}{dt}\right|;$$

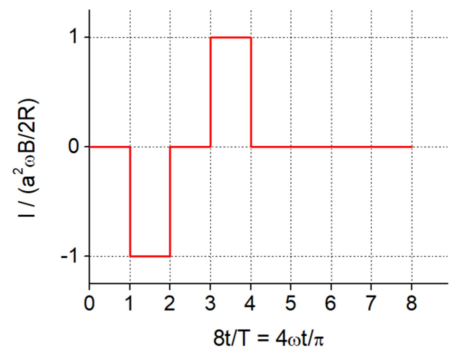
$$|\Phi_B| = (\pi a^2)\left(\frac{\omega t - \pi/4}{2\pi}\right)B \quad \Rightarrow \quad \left|\frac{d\Phi_B}{dt}\right| = \frac{\pi a^2 \omega B}{2\pi} = \frac{a^2\omega B}{2}$$

$$\boxed{I = \frac{a^2\omega B}{2R}}$$

b)

- sentido horário: + (corrente positiva);
- sentido anti-horário: - (corrente negativa)
- espira entrando na região de campo magnético: Φ_B aumenta $\Rightarrow I < 0$
- espira saindo na região de campo magnético: Φ_B diminiu $\Rightarrow I > 0$

$$T = \frac{2\pi}{\omega}$$



c)

$$P = RI^2 = \frac{a^4\omega^2B^2}{4R} \quad \text{nos trechos em que } I \neq 0$$

$$E_{dissipada} = \int_0^T P(t)\,dt = 2P\frac{T}{8} = \frac{PT}{4} = \frac{a^4\omega^2B^2}{4R}\frac{2\pi}{\omega}\frac{1}{4}$$

$$\boxed{E_{dissipada} = \frac{\pi a^4\omega B^2}{8R}}$$

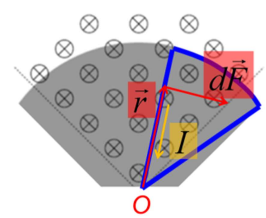
d)

$$d\vec{F} = I d\vec{\ell} \times \vec{B}$$

$d\vec{N} = \vec{r} \times d\vec{F}$ (o vetor torque entra no plano do papel)

No arco o torque é nulo. No raio $d\ell = dr$.

$$|d\vec{N}| = r dF = r\, I d\ell\, B = IBrdr$$



$$\boxed{|\vec{N}| = IB\int_0^a r dr = \frac{IBa^2}{2} = \frac{B^2a^4\omega}{4R} \qquad \vec{N} = \frac{Ba^4\omega}{4R}\vec{B}}$$

Q7.

a)

No interior do condutor, $\vec{E} = 0$ [0,1 pt] e $V = cte$.

Por continuidade, $V(r < R) = V(r = R^+) = A/R$.

b)

$$\vec{E}(\vec{r}) = -\nabla V(\vec{r}) = -\left(-\frac{A}{r^2}\hat{r}\right) = \frac{A}{r^2}\hat{r}$$

$$\vec{D}_1(\vec{r}) = \varepsilon_1 \vec{E}(\vec{r}) = \frac{\varepsilon_1 A}{r^2}\hat{r}$$

$$\vec{D}_2(\vec{r}) = \varepsilon_2 \vec{E}(\vec{r}) = \frac{\varepsilon_2 A}{r^2}\hat{r}$$

c)

Lei de Gauss para o vetor deslocamento elétrico:

$$\oint_S \vec{D}\cdot\hat{n}da = Q_\ell \quad \Rightarrow \quad \left(\vec{D}_{fora} - \vec{D}_{dentro}\right)\cdot\hat{n} = \sigma_\ell$$

$$\vec{D}_{dentro} = 0 \quad \Rightarrow \quad \sigma_\ell = \vec{D}_{fora}\cdot\hat{n}\Big|_{r=R} = \vec{D}_{fora}\cdot\hat{r}\Big|_{r=R}$$

$$\sigma_{\ell 1} = \vec{D}_1\cdot\hat{r}\Big|_{r=R} = \frac{\varepsilon_1 A}{R^2}$$

$$\sigma_{\ell 2} = \vec{D}_2\cdot\hat{r}\Big|_{r=R} = \frac{\varepsilon_2 A}{R^2}$$

d)

$$\sigma_{Pi} = \vec{P}_i\cdot\hat{n}$$

Como $\vec{P}_i \parallel \hat{r}$ e $\hat{n} \perp \hat{r}$, $\sigma_{Pi} = 0$

Q8.

a)

O elemento ij da matriz $\hat{H}$ é dado por $\langle i|\hat{H}|j\rangle$. Assim, $\hat{H}=\begin{bmatrix} E_1 & 0 & W \\ 0 & E_2 & 0 \\ W & 0 & E_1 \end{bmatrix}$.

b)

Autovalores:

$$\begin{vmatrix} E_1-\varepsilon & 0 & W \\ 0 & E_2-\varepsilon & 0 \\ W & 0 & E_1-\varepsilon \end{vmatrix}=0 \;\Rightarrow\; (E_1-\varepsilon)^2(E_2-\varepsilon)-W^2(E_2-\varepsilon)=0$$

$$\begin{cases} \varepsilon_1 = E_1+W \\ \varepsilon_2 = E_2 \\ \varepsilon_3 = E_1-W \end{cases}$$

Autovetores:

$$\hat{H}|\psi\rangle=\varepsilon|\psi\rangle; \quad |\psi\rangle=\alpha|1\rangle+\beta|2\rangle+\gamma|3\rangle$$

$\varepsilon=\varepsilon_1=E_1+W$:

$$\begin{bmatrix} E_1 & 0 & W \\ 0 & E_2 & 0 \\ W & 0 & E_1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}=(E_1+W)\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \Rightarrow \begin{cases} E_1\alpha+W\gamma=(E_1+W)\alpha \Rightarrow \alpha=\gamma \text{ qq.} \\ E_2\beta=(E_1+W)\beta \Rightarrow \beta=0 \\ (W\alpha+E_1\gamma=(E_1+W)\gamma \Rightarrow \alpha=\gamma \text{ qq.}) \end{cases}$$

$$|\psi_1\rangle=\frac{|1\rangle+|3\rangle}{\sqrt{2}}$$

$\varepsilon=\varepsilon_2=E_2$:

$$\begin{bmatrix} E_1 & 0 & W \\ 0 & E_2 & 0 \\ W & 0 & E_1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}=E_2\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \Rightarrow \begin{cases} E_1\alpha+W\gamma=E_2\alpha \Rightarrow \alpha=\gamma=0 \\ E_2\beta=E_2\beta \Rightarrow \beta \text{ qq.} \\ (W\alpha+E_1\gamma=E_2\gamma \Rightarrow \alpha=\gamma=0) \end{cases}$$

$$|\psi_2\rangle=|2\rangle$$

$\varepsilon=\varepsilon_3=E_1-W$:

$$\begin{bmatrix} E_1 & 0 & W \\ 0 & E_2 & 0 \\ W & 0 & E_1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}=E_1-W\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \Rightarrow \begin{cases} E_1\alpha+W\gamma=(E_1-W)\alpha \Rightarrow \alpha=-\gamma \text{ qq.} \\ E_2\beta=(E_1-W)\beta \Rightarrow \beta=0 \\ (W\alpha+E_1\gamma=(E_1-W)\gamma \Rightarrow \alpha=-\gamma \text{ qq.}) \end{cases}$$

$$|\psi_3\rangle=\frac{|1\rangle-|3\rangle}{\sqrt{2}}$$

c)

Na base $\{|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\}$ o elemento ij da matriz $\hat{H}_0$ é dado por $\langle\psi_i|\hat{H}_0|\psi_j\rangle$.

Partindo da base $\{|1\rangle, |2\rangle, |3\rangle\}$:

$$\hat{H}_0|\psi_1\rangle = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} E_1/\sqrt{2} \\ 0 \\ E_1/\sqrt{2} \end{bmatrix} = E_1\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = E_1|\psi_1\rangle$$

$$\hat{H}_0|\psi_2\rangle = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ E_2 \\ 0 \end{bmatrix} = E_2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = E_2|\psi_2\rangle$$

$$\hat{H}_0|\psi_3\rangle = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} E_1/\sqrt{2} \\ 0 \\ -E_1/\sqrt{2} \end{bmatrix} = E_1\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} = E_1|\psi_3\rangle$$

Assim, na base ortonormal $\{|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\}$,

$$\boxed{\hat{H}_0 = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix}}, \text{ ou } \boxed{\hat{H}_0 = E_1|\psi_1\rangle\langle\psi_1| + E_2|\psi_2\rangle\langle\psi_2| + E_1|\psi_3\rangle\langle\psi_3|}$$

d)

Possibilidades de degenerescência:

$$\varepsilon_1 = \varepsilon_2 \Rightarrow E_1 + W = E_2 \Rightarrow W = E_2 - E_1$$
$$\varepsilon_1 = \varepsilon_3 \Rightarrow E_1 + W = E_1 - W \Rightarrow W = 0$$
$$\varepsilon_2 = \varepsilon_3 \Rightarrow E_2 = E_1 - W \Rightarrow W = E_1 - E_2$$

Os valores não nulos de W que geram degenerescência são $\boxed{W = (E_2 - E_1)}$ e $\boxed{W = (E_1 - E_2)}$.

Q9.

a)

$$E = \gamma mc^2 = \gamma E_0 \;\Rightarrow\; \gamma = \frac{E}{E_0} = \frac{5\times10^{12}\,\text{eV}}{1\times10^{9}\,\text{eV}} = 5000$$

$$\gamma = \left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} \;\Rightarrow \frac{v}{c} = \left(1-\gamma^{-2}\right)^{1/2} \simeq 1-\frac{1}{2}\gamma^{-2} = 1-\Delta$$

$$\boxed{\Delta = \frac{1}{2}\gamma^{-2} = \frac{1}{2}(5000)^{-2} = 2\times10^{-8}}$$

b)

Conservação do momento:

$$\vec{P}_i = \vec{P}_f;\quad \vec{P}_i = \vec{p}_{iA} + \vec{p}_{iB} = 0 \;\Rightarrow\; \vec{P}_f = 0$$

X está em repouso em S.

Conservação da energia:

$$E_X = E_A + E_B = 2E_A$$

$$E_X = m_X c^2 \;\text{(repouso)}$$

$$\boxed{m_X = \frac{2E_A}{c^2}}$$

c)

Conservação do momento: $\gamma m_0 v_0 = \gamma' m_Y v_Y$ (1)

Conservação da energia:

$$\gamma m_0 c^2 + m_0 c^2 = \gamma' m_Y c^2 \;\Rightarrow\; (\gamma+1)m_0 = \gamma' m_Y \;\Rightarrow\; m_Y = \frac{(\gamma+1)}{\gamma'}m_0 \qquad (2)$$

$$(2) \text{ em } (1):\; \gamma m_0 v_0 = (\gamma+1)m_0 v_Y \;\Rightarrow\; v_Y = \frac{\gamma}{\gamma+1}v_0$$

$$\left.\begin{aligned} &\text{Mas}\quad \gamma'^{-2} = 1-\left(\frac{v_Y}{c}\right)^2 = 1-\frac{\gamma^2}{(\gamma+1)^2}\left(\frac{v_0}{c}\right)^2 \\ &\text{e}\quad \left(\frac{v_0}{c}\right)^2 = 1-\gamma^{-2} = \frac{\gamma^2-1}{\gamma^2} \end{aligned}\right\}\gamma'^{-2} = 1-\frac{\gamma^2}{(\gamma+1)^2}\frac{\gamma^2-1}{\gamma^2} = 1-\frac{\gamma-1}{\gamma+1} = \frac{2}{\gamma+1}$$

$$\gamma'^{-1} = \sqrt{\frac{2}{\gamma+1}}$$

$$\text{Novamente em (2):}\; m_Y = \frac{(\gamma+1)}{\gamma'}m_0 = (\gamma+1)\sqrt{\frac{2}{\gamma+1}}m_0$$

$$\boxed{m_Y = \sqrt{2(\gamma+1)}m_0}$$

Q10.

a)

Energia dos microestados para um par de íons:

$$E = -J\sigma_1\sigma_2 - \mu_B h(\sigma_1 + \sigma_2)$$

σ_1	σ_2	$E\{\sigma_1,\sigma_2\}$
$+1$	$+1$	$-J-2\mu_B h$
-1	-1	$-J+2\mu_B h$
$+1$	-1	$+J$
-1	$+1$	$+J$

$$Z_{par} = \sum_{\{\sigma_1,\sigma_2\}} e^{-\beta E\{\sigma_1,\sigma_2\}} = e^{-\beta(-J-2\mu_B h)} + e^{-\beta(-J+2\mu_B h)} + 2e^{-\beta J} = 2e^{-\beta J} + e^{\beta J}\left(e^{2\beta\mu_B h} + e^{-2\beta\mu_B h}\right)$$

$$\boxed{Z_{par} = 2\left[e^{-\beta J} + e^{\beta J}\cosh(2\beta\mu_B h)\right]}$$

b)

Para 2 pares de íons:

$$Z_{2\,pares} = \sum_{\{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}} e^{-\beta E\{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}} = \sum_{\{\sigma_1,\sigma_2,\sigma_3,\sigma_4\}} e^{-\beta\left[E\{\sigma_1,\sigma_2\}+E\{\sigma_3,\sigma_4\}\right]} =$$

$$= \sum_{\{\sigma_1,\sigma_2\}} e^{-\beta E\{\sigma_1,\sigma_2\}} \sum_{\{\sigma_3,\sigma_4\}} e^{-\beta E\{\sigma_3,\sigma_4\}} = \left(\sum_{\{\sigma_1,\sigma_2\}} e^{-\beta E\{\sigma_1,\sigma_2\}}\right)^2 = Z_{par}^2$$

Para N/2 pares de íons:

$$\boxed{\begin{aligned} Z_{N/2\,pares} &= \left(Z_{par}\right)^{N/2} \\ Z_{N/2\,pares} &= \left\{2\left[e^{-\beta J} + e^{\beta J}\cosh(2\beta\mu_B h)\right]\right\}^{N/2} \end{aligned}}$$

c)

$$M = k_B T\frac{\partial \ln Z}{\partial h} = \frac{N}{2}k_B T\frac{\partial}{\partial h}\left(\ln\left\{2\left[e^{-\beta J} + e^{\beta J}\cosh(2\beta\mu_B h)\right]\right\}\right)$$

$$M = \frac{N}{2}k_B T\frac{2e^{\beta J}\operatorname{senh}(2\beta\mu_B h)2\beta\mu_B}{2\left[e^{-\beta J} + e^{\beta J}\cosh(2\beta\mu_B h)\right]}$$

$$\boxed{M = N\mu_B\frac{e^{\beta J}\operatorname{senh}(2\beta\mu_B h)}{\left[e^{-\beta J} + e^{\beta J}\cosh(2\beta\mu_B h)\right]}}$$

d)

$$\boxed{\text{Como}\quad \sinh(0) = 0,\quad \lim_{h\to 0} M = 0}$$

Não pode representar um material ferromagnético, pois não há magnetização residual (ou espontânea).