AGA0503 - EXERCÍCIO DE PROGRAMAÇÃO 2

EXERCÍCIO 4

→ Resolver o sistema $\begin{bmatrix} 18 & -9 & -3 \\ 3 & 15 & -9 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 8 \\ 4 \end{bmatrix}$ por decomposição LU a partir do algoritmo de Crout

▷ $\begin{bmatrix} 18 & -9 & -3 \\ 3 & 15 & -9 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

▷ Para $i = 1, \dots, j$: $u_{ij} = a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj}$

Então $\boxed{\begin{aligned} u_{11} &= 18 \\ u_{12} &= -9 \\ u_{13} &= -3 \end{aligned}}$

$u_{22} = 15 + 9 l_{21}$

$u_{23} = -9 + 3 l_{21}$

$u_{33} = -3 + 3 l_{31} - (-9 + 3 l_{21}) l_{32}$

▷ Para $i = j+1, \dots, n$: $l_{ij} = \frac{1}{u_{jj}} \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \right)$

Então $l_{21} = \frac{3}{18} \Rightarrow \boxed{l_{21} = \frac{1}{6}} \Rightarrow u_{22} = 15 + \frac{3}{2} \Rightarrow \boxed{u_{22} = \frac{33}{2}}$

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$u_{23} = -9 + \frac{1}{2} \Rightarrow \boxed{u_{23} = -\frac{17}{2}}$

$\boxed{l_{31} = \frac{1}{18}}$ $\quad l_{32} = \frac{2}{33} \left(1 - \frac{1}{18} (-9) \right) = \frac{2}{33} \cdot \frac{3}{2}$

$\boxed{l_{32} = \frac{1}{11}}$

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$$u_{33} = -3 + \frac{1}{6} - \left(-9 + \frac{1}{2}\right)\frac{1}{11} = -\frac{17}{6} + \frac{17}{2}\cdot\frac{1}{11} = \frac{11}{6}\cdot 31$$

$$\boxed{u_{33} = \frac{187}{3}}$$

▷ $Ax = b \Rightarrow (LU)x = b \Rightarrow L(Ux) = b \Rightarrow Ly = b$
$Ux = y$

▷ Resolvendo $Ly = b$: $y_1 = \frac{b_1}{l_{11}}$ e $y_i = \frac{1}{l_{ii}}\left[b_i - \sum_{j=1}^{i} l_{ij} y_i\right]$
$i = 2, 3, \dots, n$

$\boxed{y_1 = 13}$ $\quad y_2 = 8 - \sum_{j=1}^{2} l_{2j} y_2 = 8 - (l_{21} + l_{22}) y_2$

$$y_2 = \frac{8}{1 + 1/6 + 1} \Rightarrow \boxed{y_2 = \frac{48}{13}}$$

$$y_3 = 4 - \sum_{j=1}^{3} l_{3j} y_3 = 4 - y_3(l_{31} + l_{32} + 1) \Rightarrow y_3 = \frac{4}{2 + 1/18 + 1/11} =$$

$$= \frac{792}{396 + 11 + 18} \Rightarrow \boxed{y_3 = \frac{792}{425}}$$

▷ Resolvendo $Ux = y$: $x_n = \frac{y_n}{u_{nn}}$ e $x_i = \frac{1}{u_{ii}}\left[y_i - \sum_{j=i+1}^{n} u_{ij} x_j\right]$
$i = n-1, n-2, \dots, 1$

$$x_3 = \frac{y_3}{u_{33}} \Rightarrow x_3 = \frac{792 \cdot 3}{425 \cdot 187} \Rightarrow \boxed{x_3 = \frac{2376}{79475}}$$

$$x_2 = \frac{2}{33}\left(\frac{48}{13} + \frac{17}{\cancel{2}}\,\frac{\cancel{792}\cdot 3}{425 \cdot 187}\right) = \frac{2}{33}\left(\frac{48}{13} + \frac{17 \cdot 396 \cdot 3}{425 \cdot 187}\right) =$$

$$= \frac{2}{33} \cdot \frac{18 \cdot 425 \cdot 187 + 17 \cdot 13 \cdot 396 \cdot 3}{13 \cdot 425 \cdot 187} \Rightarrow \boxed{x_2 \simeq 0{,}239}$$

$$x_1 = \frac{1}{18}\left(13 - \sum_{j=2}^{3} u_{1j}\, x_j\right) = \frac{1}{18}\left(13 - (-9)\,0{,}239 - (-3)\,\frac{2376}{79475}\right) =$$

$$= \frac{1}{18}\left(13 + 2{,}151 + 0{,}090\right) = \frac{15{,}241}{18} \Rightarrow \boxed{x_1 \simeq 0{,}847}$$