* Teorema da invariância

→ Se S_1 e S_2 são bases de V.

$$\left.\begin{array}{l} S_1 : \{v_1, \ldots, v_n\} \\ S_2 : \{w_1, \ldots, w_m\} \end{array}\right\} \boxed{n = m}$$

→ $Dim(V) = \#$ de elementos de uma base V.

α sendo uma base $B = \{v_1, \dots, v_n\}$

$D \in V = [B] V \Rightarrow \forall v \in V \; \exists$

$\alpha_1, \dots, \alpha_n \in \mathbb{R}$, tq

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

O fato de B ser um conjunto L.I. $\Rightarrow$

$\Rightarrow \forall v \in V$ os $\alpha_1, \dots, \alpha_n \in \mathbb{R}$ com a propriedade que

PERDI i.i

$\beta_1, \dots, \beta_n$

$v = \beta_1 v_1 + \dots + \beta_n v_n$

$0 = \underbrace{(\alpha_1 - \beta_1)}_{\lambda_1} v_1 + \underbrace{(\alpha_2 - \beta_2)}_{\lambda_2} v_2 + \dots + \underbrace{(\alpha_n - \beta_n)}_{\lambda_n} v_n$

$\Rightarrow \lambda_1 = \lambda_2 = \dots = \lambda_n = 0$

$\Rightarrow \alpha_1 = \beta_1, \alpha_2 = \beta_2, \dots, \alpha_n = \beta_n$

~~Calc. de dim. de um Espaço vetorial~~

$V = \mathbb{R}^n$

$\dim V = n$

$B = \{e_1, \dots, e_n\}$

$e_1 = (1, 0, \dots, 0)$
$e_2 = (0, 1, 0, \dots, 0)$
$\vdots$
$e_k = (0, 0, \dots, 0, 1, 0, \dots, 0)$
$\vdots$
$e_n = (0, \dots, 0, 1)$

$[B] = \mathbb{R}^n$

$v = (x_1, x_2, \dots, x_n) = x_1 e_1, x_2 e_2 + \dots + x_n e_n$

base canônica
$$\begin{cases} x_1 e_1 = (x_1, 0, \dots, 0) \\ \vdots \\ x_n e_n = (0, \dots, x_n) \end{cases}$$

B é L.I.

$\lambda_1 e_1 + \dots + \lambda_n e_n = 0$

$(\lambda_1, \lambda_2, \dots, \lambda_n) = (0, \dots, 0)$

$\lambda_1 = \lambda_2 = \lambda_n = 0$

Problema Típico

$w_1, \dots, w_k \in \mathbb{R}^n$

$V = [w_1, \dots, w_k]$

$\dim V = ?$

P/ determinar se a base é LI ou LD:

1. A ordem dos elementos não é relevante:

$[w_1, \dots, w_i, \dots, w_j, \dots, w_k] =$

$= [w_1, \dots, w_j, \dots, w_i, \dots, w_k]$

2. $[w_1, \dots, w_{i-1}, w_i + \sum_{j \neq i} \lambda_j w_j + w_{i+1}, \dots, w_k] = [w_1, \dots, w_k]$

como linear de todos os elementos da família

GENTE! x.x

$\hookrightarrow \alpha_1 w_1 + \dots + \alpha_k w_k =$
$= \beta_1 w_1 + \dots + \beta_i (w_i + \sum_{j \neq i} \lambda_j w_j) + \dots +$
$+ \beta_k w_k$

$w_1 (\beta_1 + \beta_i \lambda_1)$

$w_i (\beta_i)$

$w_2 (\beta_2 + \beta_i \lambda_2)$

$w_s (\beta_s + \beta_i \lambda_s)$

Tô entendendo nada!

$w_i = (a_{i1}, a_{i2}, \dots, a_{in})$ $i = 1, \dots, k$

$w_1 = (a_{11}, a_{12}, \dots, a_{1n})$

$w_2 = (a_{21}, a_{22}, \dots, a_{2n})$

$\vdots$

$w_k = (a_{k1}, a_{k2}, \dots, a_{kn})$

3. Chamo uma família destes vetores escalonados:

Se $w_1, \dots, w_k$ são uma família escalonada e $w_i \neq 0 \ \forall i \Rightarrow w_1, \dots, w_k$ são LI.

$\lambda_1 w_1 + \lambda_2 w_2 + \dots + \lambda_k w_k = 0$

$\lambda w_1 = (0, \dots, 0, \lambda, a \dots$

Tendi nada e ñ consegui copiar!

Exemplo: ♡

$\mathbb{R}^4$

$w_1 = (2, 1, 1, 0)$

$w_2 = (1, 0, 1, 2)$

$w_3 = (0, -1, 1, 4)$

$\dim [w_1, w_2, w_3]$

$$\Rightarrow \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & -1 & 1 & 4 \end{pmatrix} \xrightarrow{\times 1/2} \begin{pmatrix} 1 & 1/2 & 1/2 & 0 \\ 0 & -1/2 & 1/2 & 2 \\ 0 & -1 & 1 & 4 \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Então $\dim [w_1, w_2, w_3] = 2$

V esp. vetorial

$W_1, W_2 \subset V$ subespaços

$W_1 \cap W_2$ é um subespaço de V

$W_1 + W_2$ soma de subespaços

$W_1 + W_2 = \{ v_1 + v_2 \mid v_1 \in W_1, v_2 \in W_2 \}$

$= [W_1 \cup W_2]$

$\dim (W_1 + W_2) \neq \dim (W_1) + \dim (W_2)$

Se $W_1 = W_2 \Rightarrow \boxed{W_1 + W_2 = W_1 = W_2}$

$a_1, \dots, a_s$ base de W_1

$\dim |W_1| = s$

$b_1, \dots, b_r$ Base de W_2

$\dim |W_2| = r$

$[\underbrace{a_1, \dots, a_s, b_1, \dots, b_r}_{s+r}] = W_1 + W_2$

$\dim (W_1 + W_2) \leq s + r = \dim (W_1) + \dim (W_2)$

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) -$$
$$- \dim(W_1 \cap W_2)$$

$$\dim(W_1 + W_2) + \dim(W_1 \cap W_2) =$$
$$= \dim(W_1) + \dim(W_2)$$