UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE MATEMATICA E ESTATISTICA

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Nota

Nome_____

Data ___/___/___ Nº. USP_____

Disciplina_____ Turma_____ Prof._____

Q 1
Q 2
Q 3
Q 4

(Q1) $\int_0^{\pi} \underbrace{e^{\cos x}}_{f(x)} dx$

$$|f'''(x)| = |e^{\cos x}(\operatorname{sen}^4 x - 4\operatorname{sen}^2 x + 3\cos^2 x + \cos x - 6\operatorname{sen}^2 x \cos x)|$$

$$\leq e^1 \cdot |\operatorname{sen}^4 x - 4\operatorname{sen}^2 x + 3\cos^2 x + \cos x - 6\operatorname{sen}^2 x \cos x|$$

$$\leq e \cdot (\underbrace{|\operatorname{sen}^4 x|}_{\leq 1} + \underbrace{|-4\operatorname{sen}^2 x|}_{\leq 4} + \underbrace{|3\cos^2 x|}_{\leq 3} + \underbrace{|\cos x|}_{\leq 1} + \underbrace{|-6\operatorname{sen}^2 x \cos x|}_{\leq 6})$$

$$\leq 15e < 41$$

erro de n-Simpson $\leq \frac{41}{2880} \cdot \pi^5 \cdot n^{-4} < 4.36\, n^{-4}$

Impomos $4.36\, n^{-4} < 10^{-2}$

$n^4 > 436$

$n > \sqrt[4]{436} \cong 4.56$

$\boxed{n \geq 5}$

Escolha da casa decimal de arredondamento:

erro total (valores de f e arredondamento final) : $\frac{1+(b-a)}{2} \times 10^{-N}$

$\approx 2 \times 10^{-N}$

Com $N=4$ esse erro fica em $2 \times 10^{-4} < 10^{-2}$.

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① continuação 5 - Simpson (5 células, 2×5+1 pontos)

	x	f(x)
I_1	0	2.7183 × 1
	$\pi/10$	2.5884 × 4
	$2\pi/10$	2.2457 × 2
I_2	$3\pi/10$	1.8000 × 4
	$4\pi/10$	1.3621 × 2
I_3	$5\pi/10$	1.0000 × 4
	$6\pi/10$	0.7342 × 2
I_4	$7\pi/10$	0.5556 × 4
	$8\pi/10$	0.4453 × 2
I_5	$9\pi/10$	0.3863 × 4
	π	0.3679 × 1

$$5\text{-Simpson} = \pi \cdot \frac{37.9820}{6 \times 5}$$

$$= 3.977465739$$

(último arredondamento)

$$= 3.9775$$

253212
+ 95746
+ 30862
37.9820

R: 3.9775 ± 10^{-2}

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(2)

(a)

x	$f(x)$		
0	1	$\frac{1}{a}$	
a	2		$\frac{-2}{1-a} - \frac{1}{a}$
		$\frac{-2}{1-a}$	
1	0		

$$\boxed{P_a(x) = 1 + \frac{1}{a} \cdot x + \left(\frac{-2}{1-a} - \frac{1}{a}\right) x(x-a)}$$

(b) $$P_a'(x) = \frac{1}{a} + \left(\frac{-2}{1-a} - \frac{1}{a}\right)(2x-a)$$

$$P_a'\left(\tfrac{1}{2}\right) = \frac{1}{a} + \left(\frac{-2}{1-a} - \frac{1}{a}\right)(1-a)$$

$$= \frac{1}{a} + \left(-2 - \frac{1-a}{a}\right) = \frac{1}{a} - 2 - \frac{1}{a} + \frac{a}{a} = -1$$

$P_a'\left(\frac{1}{2}\right) = -1$, independentemente de a.

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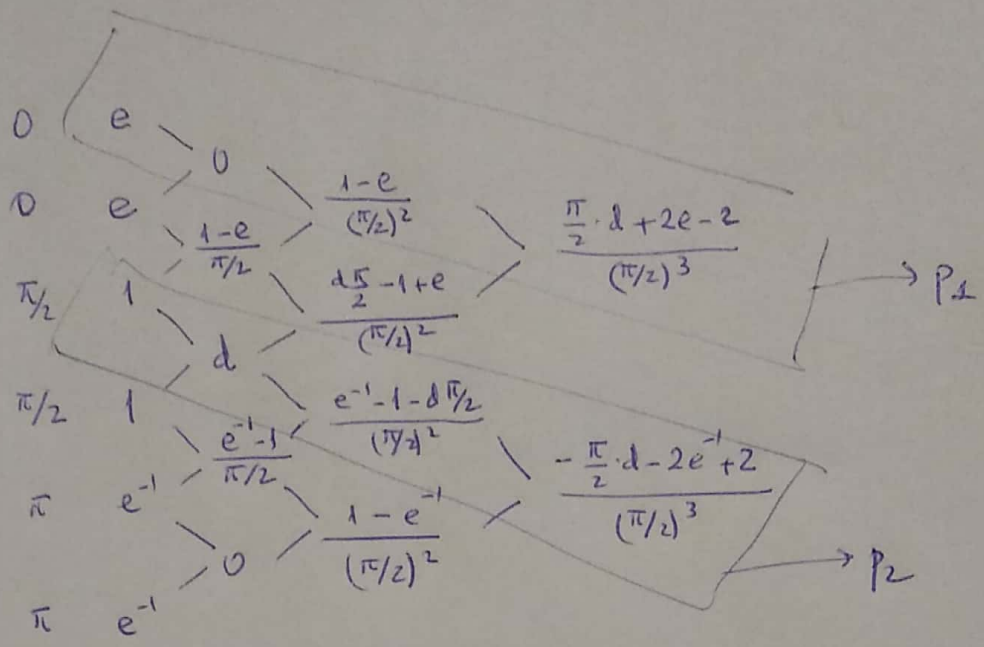
(3)

fundindo os dois Hermite de grau 3



$$P_1(x) = e + 0\cdot x + \frac{1-e}{(\pi/2)^2}x^2 + \frac{\frac{\pi}{2}d + 2e - 2}{(\pi/2)^3}x^2\left(x - \frac{\pi}{2}\right)$$

$$P_2(x) = 1 + d\left(x - \frac{\pi}{2}\right) + \frac{e^{-1} - 1 - \frac{\pi}{2}d}{(\pi/2)^2}\left(x - \frac{\pi}{2}\right)^2 + \frac{-\frac{\pi}{2}d - 2e^{-1} + 2}{(\pi/2)^3}\left(x - \frac{\pi}{2}\right)^2(x - \pi)$$

$$P_1''(x) = \frac{1}{(\pi/2)^3}\left\{\pi(1-e) + \left(\frac{\pi}{2}d + 2e - 2\right)(6x - \pi)\right\}$$

$$\rightsquigarrow P_1''\left(\frac{\pi}{2}\right) = \frac{1}{(\pi/2)^3}\left[\pi(1-e) + 2\pi\left(\frac{\pi}{2}d + 2e - 2\right)\right]$$

$$P_2''(x) = \frac{1}{(\pi/2)^3}\left\{\pi\left(e^{-1} - 1 - \frac{\pi}{2}d\right) + \left(-\frac{\pi}{2}d - 2e^{-1} + 2\right)(6x - 4\pi)\right\}$$

$$\rightsquigarrow P_2''\left(\frac{\pi}{2}\right) = \frac{1}{(\pi/2)^3}\left[\pi\left(e^{-1} - 1 - \frac{\pi}{2}d\right) - \pi\left(-\frac{\pi}{2}d - 2e^{-1} + 2\right)\right]$$

$$P_1''\left(\frac{\pi}{2}\right) = P_2''\left(\frac{\pi}{2}\right) \Rightarrow \pi(1-e) + 2\pi\left(\frac{\pi}{2}d + 2e - 2\right) = \pi\left(e^{-1} - 1 - \frac{\pi}{2}d\right) + \pi\left(\frac{\pi}{2}d + 2e^{-1} - 2\right)$$

$$1 - e + \pi d + 4e - 4 = e^{-1} - 1 - \frac{\pi}{2}d + \frac{\pi}{2}d + 2e^{-1} - 2$$

$$-3 + 3e + \pi d = 3e^{-1} - 3 \Rightarrow \boxed{d = -\frac{3}{\pi}(e - e^{-1})}$$

$(\approx -2{,}244)$

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(4)

Fazendo Hermite por partes com $d = f'(\frac{\pi}{2})$:

$$f'(\theta) = -\operatorname{sen}\theta\, e^{\cos\theta}$$

$$f'(\tfrac{\pi}{2}) = -1 \cdot e^{0} = \underline{\underline{-1}}$$

Basta colocar $\underline{d = -1}$ em P_1 e P_2 do exercício anterior.

$$P_1(x) = e + \frac{1-e}{(\pi/2)^2}\, x^2 + \frac{2e-2-\pi/2}{(\pi/2)^3}\, x^2\left(x-\frac{\pi}{2}\right)$$

$$P_2(x) = 1 + \frac{\pi}{2} - x + \frac{e^{-1}-1+\frac{\pi}{2}}{(\pi/2)^2}\left(x-\frac{\pi}{2}\right)^2 + \frac{\frac{\pi}{2}-2e^{-1}+2}{(\pi/2)^3}\left(x-\frac{\pi}{2}\right)^2(x-\pi)$$

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