GABARITO PII 21/06/2017

(1) $h(w) = \int_0^{w^3} \operatorname{sen}(w t^2)\,dt$

$\varphi(x,y) = \int_0^{x} \operatorname{sen}(y t^2)\,dt$

$\frac{\partial \varphi}{\partial x} = \frac{d}{dx} \int_0^{x} \operatorname{sen}(y t^2)\,dt$ para y const.

$= \operatorname{sen}(y x^2)$ (TFC.)

$\frac{\partial \varphi}{\partial y} = \frac{d}{dy} \int_0^{x} \operatorname{sen}(y t^2)\,dt = \int_0^{x} \frac{\partial}{\partial y} \operatorname{sen}(y t^2)\,dt$

$= \int_0^{x} \cos(y t^2)\, t^2\,dt$ (dif. embaixo do integral)

Para $\gamma(w) = (w^3, w)$ temos

$h(w) = \varphi(\gamma(w))$

②

Daí, $h'(w) = (\Delta \varphi)(\gamma(w)) \cdot \gamma'(w)$

(Regra da Cadeia)

$$\Delta \varphi = \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}\right)$$

$$\Delta \varphi(x,y) = \left(\operatorname{sen}(yx^2), \int_0^x \cos(yt^2)t^2\,dt\right)$$

$$\Delta \varphi(\gamma(w)) = \left(\operatorname{sen}(w(w^3)^2), \int_0^{w^3} \cos(wt^2)t^2\,dt\right)$$

$$h'(w) = \left(\operatorname{sen}(w^7), \int_0^{w^3} \cos(wt^2)t^2\,dt\right) \cdot \underbrace{(3w^2, 1)}_{\gamma'(w)}$$

$$= \boxed{3w^2 \operatorname{sen}(w^7) + \int_0^{w^3} t^2 \cos(wt^2)\,dt}$$

2 $\int_\gamma x^{17}+y^3\,dx - x^3+y^{35}\,dy$

$= \int_\gamma P\,dx + Q\,dy = \int_\gamma F\cdot d\gamma$

onde $F=(P,Q)$ e $P(x,y)=x^{17}+y^3$
$Q(x,y)=-x^3+y^{35}$

γ é curva fechada, fronteira do disco unitário Ω, então pelo Green:

$\int_\gamma F\cdot d\gamma = \iint_\Omega \text{rot}\,F\cdot k\,dx\,dy$

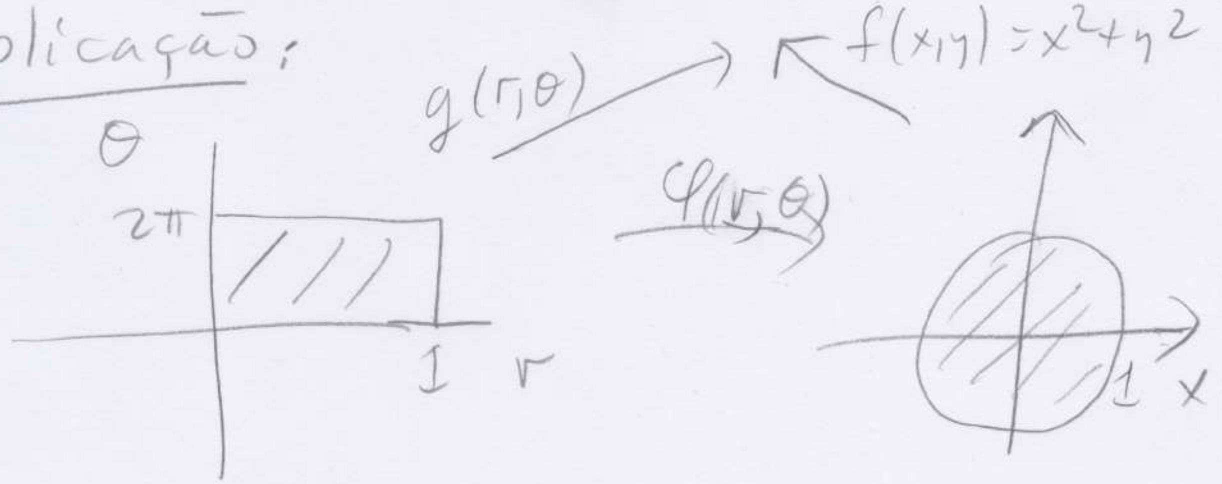
$\text{rot}\,F\cdot k = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -3x^2-3y^2$

$-3\iint_\Omega x^2+y^2\,dx\,dy = -3\int_{\theta=0}^{2\pi}\int_{r=0}^{1} r^2\,r\,dr\,d\theta$

$$= -3 \int_{r=0}^{1} r^3 \left(\int_0^{2\pi} d\theta \right) dr$$

$$= -3(2\pi) \left(\frac{r^4}{4} \right) \Big|_0^1 = -3(2\pi)\frac{1}{4} = \boxed{\frac{-3}{2}\pi}$$

Explicação:



$$\varphi(r,\theta) = (x,y) = (r\cos\theta, r\,\text{sen}\,\theta)$$

$$g(r,\theta) = f(\varphi(r,\theta)) = (r\cos\theta)^2 + (r\,\text{sen}\,\theta)^2$$

$$= r^2$$

$$D\varphi = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\,\text{sen}\,\theta \\ \text{sen}\,\theta & r\cos\theta \end{bmatrix}$$

$$|D\varphi| = r\cos^2\theta + r\,\text{sen}^2\theta = r$$

$$\int_0^{2\pi}\int_0^1 f\circ\varphi(r,\theta)\,|D\varphi(r,\theta)|\,dr\,d\theta = \iint_\Omega f(x,y)\,dx\,dy$$

3

a) $F(x,y) = (P, Q)$ $\quad P(x,y) = 0$

$Q(x,y) = x$

$$\int_\gamma F \cdot d\gamma = \iint_\Omega \operatorname{rot} F \cdot k \, dx\, dy \quad (\underline{\text{Green}})$$

$$\operatorname{rot} F \cdot \underline{k} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$$

então $\displaystyle\int_\gamma F \cdot d\gamma = \iint_\Omega 1 \, dx\, dy = \text{área}(\Omega)$

b) $G(x,y) = (P, Q)$ $\quad P(x,y) = -y$

$Q(x,y) = 0$

$$\int_\gamma G \cdot d\gamma = \iint_\Omega \operatorname{rot} G \cdot k \, dx\, dy$$

$$\operatorname{rot} G = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$$

<u>então</u> $\displaystyle\int_\gamma G \cdot d\gamma = \text{área}(\Omega)$ também!

(b)

$$\int_\gamma -y\,dx + x\,dy \qquad H(x,y) = (P, Q)$$

$P = -y$

$Q = x$

$$= \iint_\Omega \text{rot}\, H \cdot \underline{k}\, dx\,dy = \iint_\Omega 2\, dx\,dy = \boxed{2 \cdot \text{área}}$$

$$\text{rot}\, H = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 + 1 = 2$$

Também, $\displaystyle\int_\gamma F \cdot d\gamma = \int_\gamma x\,dy = \text{área}(\Omega)$

$$\int_\gamma G \cdot d\gamma = \int_\gamma -y\,dx = \text{área}(\Omega)$$

$$\int_\gamma -y\,dx + x\,dy = \int_\gamma -y\,dx + \int_\gamma x\,dy = \boxed{2 \cdot \text{área}}$$

3c) Não é conservativo, pois $\displaystyle\int_\gamma H \cdot d\gamma \neq 0$

Ou, pois rot $H \neq \underline{0}$

e conservativo $\Rightarrow$ rot $= \underline{0}$.

4 $F(x,y) = (x,y) = (P,Q)$ $\quad P(x,y) = x$

$Q(x,y) = y$

4a $\operatorname{div} F = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y}$

$= 1 + 1 = 2$

$\operatorname{rot} F = \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right) \underline{k} = \underline{0}$

4b Fluxo $= \displaystyle\int_\gamma F \cdot \underline{n}\, ds = \iint_\Omega \operatorname{div} F \, dx dy$

pela Teorema da Divergencia

$= \displaystyle\iint_\Omega 2\, dx dy = 2 \cdot \text{área}(\Omega)$

Dado Fluxo $= 5 = 2 \cdot \text{área}(\Omega)$ então

$\boxed{\text{Área}(\Omega) = \dfrac{5}{2}}$

5 a

8

$$\varphi(x,y) = \arctan\left(\frac{y}{x}\right) \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{\partial \varphi}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \underbrace{\frac{\partial}{\partial x}\left(\frac{y}{x}\right)}$$

$$\frac{0 \cdot x - y \cdot 1}{x^2} = -\frac{y}{x^2}$$

$$= \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$\frac{\partial \varphi}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \underbrace{\frac{\partial}{\partial y}\left(\frac{y}{x}\right)}_{\frac{1}{x}} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$$

$$= \frac{x}{x^2+y^2}$$

$$\nabla \varphi = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right) = F(x,y)$$

(9)

(5b) $\boxed{F=\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)}$ $\gamma(t)=(t,t)$

$\gamma'(t)=(1,1)$

$$\int_\gamma F\cdot d\gamma = \int_1^2 F(\gamma(t))\cdot\gamma'(t)\,dt$$

$$F(\gamma(t)) = \left(\frac{-t}{t^2+t^2}, \frac{t}{t^2+t^2}\right) = \left(\frac{-1}{2t}, \frac{1}{2t}\right)$$

$\gamma'(t)=(1,1)$

$$\int_\gamma F\cdot d\gamma = \int_1^2 \left(\frac{-1}{2t}, \frac{1}{2t}\right)\cdot(1,1)\,dt$$

$$= \int_1^2 -\frac{1}{2t}+\frac{1}{2t}\,dt = \boxed{0}$$

(5c) $\eta(t)=(1,t)$ $0\le t\le 1$ $\eta'(t)=(0,1)$

$$\int_\eta F\cdot d\eta = \int_0^1 \left(\frac{-t}{1+t^2}, \frac{1}{1+t^2}\right)\cdot(0,1)\,dt$$

$$= \int_0^1 \frac{1}{1+t^2}\,dt = \arctan(t)\Big|_0^1$$

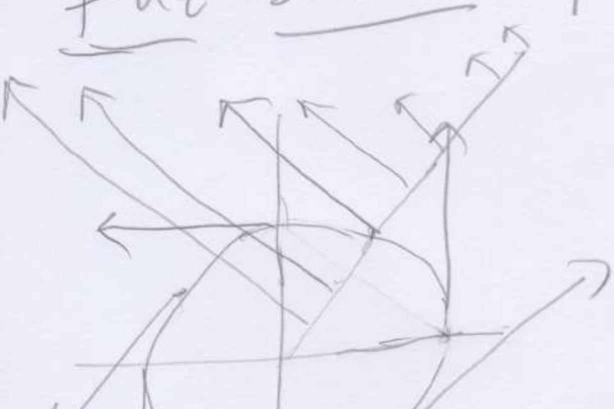
$$= \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$$

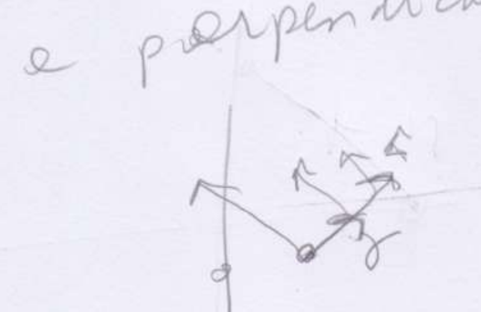
$\tan\frac{\pi}{4} = 1$

$\tan 0 = 0$

Que faz sentido pois o campo é:

e o caminho γ e perpendicular isto





enquanto η conta positivamente.