a) $\oint_c \vec{B}\cdot d\vec{r} = \mu_0 I_{int}$ $\qquad I_{int} = \iint_S \vec{j}\cdot\hat{n}\, ds$

Pela lei de Ohm microscópica, temos que:

$$\vec{E} = \rho\cdot\vec{j} \Rightarrow \vec{j} = \frac{\vec{E}}{\rho} \Rightarrow \vec{j} = \frac{E_0}{\rho}\hat{k}$$

Para $r \leq a$:

$$\oint_{C_1} \vec{B}\cdot d\vec{r} = \mu_0 I_{int} \qquad (1)$$

$$\oint_{C_1} \vec{B}\cdot d\vec{r} = \int_0^{2\pi} B r\, d\theta = 2\pi r B(r) \qquad (2)$$

$$\mu_0 I_{int} = \mu_0 \iint_{S_1} \vec{j}\cdot\hat{n}\, ds = \mu_0 \int_0^{2\pi}\int_0^{r} \frac{E_0}{\rho}\cdot r'\, dr'\, d\theta$$

$$\mu_0 I_{int} = \mu_0 \int_0^{2\pi}\int_0^{r} \frac{E_0}{\rho_0}\left[2+\cos\left(\frac{\pi r'}{a}\right)\right] r'\, dr'\, d\theta =$$

$$= \frac{\mu_0 E_0}{\rho_0}\cdot 2\pi \int_0^{r}\left[2+\cos\left(\frac{\pi r'}{a}\right)\right] r'\, dr' = \frac{\mu_0 E_0}{\rho_0} 2\pi\left(\frac{2r^2}{2} + \int_0^{r} \cos\left(\frac{\pi r'}{a}\right) r'\, dr'\right)$$

Utilizando a dica:

$$\frac{d}{dr'}\left[r'^n \operatorname{sen}(\alpha r')\right] = n r'^{n-1} \operatorname{sen}(\alpha r') + r'^n \cos(\alpha r')\cdot\alpha$$

Integrando ambos os lados, obtemos:

$$r'^n \operatorname{sen}(\alpha r') = \int_0^{r} n r'^{n-1} \operatorname{sen}(\alpha r')\, dr' + \int_0^{r} r'^n \cos(\alpha r')\cdot\alpha\, dr'$$

$$\int_0^{r} r'^n \cos(\alpha r')\, dr' = \frac{r'^n}{\alpha}\operatorname{sen}(\alpha r') - \frac{n}{\alpha}\int_0^{r} r'^{n-1} \operatorname{sen}(\alpha r')\, dr'$$

Para $n = 1$:

$$\int_0^{r} r' \cos(\alpha r')\alpha\, dr' = \frac{r'}{\alpha}\operatorname{sen}(\alpha r') + \frac{1}{\alpha^2}\cos(\alpha r')\Big|_0^{r}$$

Substituindo na integral:

$$\int_0^r \cos\left(\frac{\pi}{a} r'\right) r' dr' = \frac{r}{\pi} a \,sen\left(\frac{\pi r}{a}\right) + \frac{a^2}{\pi^2}\cos\left(\frac{\pi r}{a}\right) - \frac{a^2}{\pi^2}$$

Logo:

$$\mu_0 I_{int} = \frac{\mu_0 E_0}{\rho_0}\left[2\pi r^2 + 2 r a \,sen\left(\frac{\pi r}{a}\right) + \frac{2a^2}{\pi}\cos\left(\frac{\pi r}{a}\right) - \frac{2a^2}{\pi}\right] \quad (3)$$

Substituindo (3) e (2) em (1), temos:

$$2\pi r B(r) = \frac{\mu_0 E_0}{\rho_0}\left[2\pi r^2 + 2 r a \,sen\left(\frac{\pi r}{a}\right) + \frac{2a^2}{\pi}\cos\left(\frac{\pi r}{a}\right) - \frac{2a^2}{\pi}\right]$$

$$\vec{B}(r) = \frac{\mu_0 E_0}{\rho_0}\left(r + \frac{a \,sen(\pi r/a)}{\pi} + \frac{a^2}{\pi^2 r}\cos(\pi r/a) - \frac{a^2}{\pi^2 r}\right)\hat{\theta}$$

Para $a \leq r < b$:

$$\oint_{C2} \vec{B}\cdot d\vec{r} = \mu_0 I_{int} \quad (1)$$

$$\oint_{C2} \vec{B}\cdot d\vec{r} = \int_0^{2\pi} B(r)\cdot r\, d\theta = 2\pi r B(r) \quad (2)$$

$$\mu_0 I_{int} = \mu_0 \iint_{S2} \vec{J}\cdot\hat{n}\, ds = \mu_0 \int_0^{2\pi}\int_0^a \frac{E_0}{\rho_0}\left[2 + \cos\left(\pi \frac{r}{a}\right)\right] r' dr' d\theta +$$

$$+ \mu_0 \int_0^{2\pi}\int_a^r \frac{E_0}{\rho_0} r' dr' d\theta = \frac{\mu_0 E_0}{\rho_0}\cdot 2\pi\left[a^2 - \frac{2a^2}{\pi^2}\right] + \frac{\mu_0 E_0}{\rho_0}\cdot 2\pi\left[\frac{r^2}{2} - \frac{a^2}{2}\right] \quad (3)$$

Substituindo (2) e (3) em (1):

$$2\pi r B(r) = \frac{\mu_0 E_0}{\rho_0} 2\pi\left[a^2 - \frac{2a^2}{\pi^2}\right] + \frac{\mu_0 E_0}{\rho_0}\cdot 2\pi\left[\frac{r^2}{2} - \frac{a^2}{2}\right]$$

$$2\pi r B(r) = \frac{\mu_0 E_0}{\rho_0}\cdot 2\pi\left[a^2 - \frac{2a^2}{\pi^2} + \frac{r^2}{2} - \frac{a^2}{2}\right]$$

$$2\pi r B(r) = \frac{\mu_0 E_0}{\rho_0} 2\pi\left[\frac{a^2}{2} + \frac{r^2}{2} - \frac{2a^2}{\pi^2}\right] = \frac{\mu_0 E_0}{2\rho_0}\left[r + \frac{a^2}{r}\left(1 - \frac{4}{\pi^2}\right)\right]$$

$$\vec{B}(r) = \frac{\mu_0 E_0}{2\rho_0}\left[r + \frac{a^2}{r}\left(1 - \frac{4}{\pi^2}\right)\right]\hat{\theta}$$

Para $r \geqslant b$:

$$\oint_{C_3} \vec{B} \cdot d\vec{r} = \mu_0 I_{int} \quad (1)$$

$$\oint_{C_3} \vec{B} \cdot d\vec{r} = 2\pi r B \, (2)$$

$$\mu_0 I_{int} = \mu_0 \int_0^{2\pi} \int_0^{b} \frac{E_0}{\rho} r' \, dr' \, d\theta = \mu_0 E_0 \cdot 2\pi \int_0^{b} \frac{r'}{\rho} \, dr' =$$

$$= \mu_0 E_0 2\pi \left[\int_0^{a} \frac{r'}{\rho_0} \left[2 + \cos\left(\frac{\pi}{a} r'\right) \right] dr' + \int_a^{b} \frac{r'}{\rho_0} \, dr' \right]$$

$$= \frac{\mu_0 E_0 2\pi}{\rho_0} \left[a^2 - \frac{2a^2}{\pi^2} + \frac{b^2 - a^2}{2} \right]$$

Substituindo (2) e (3) em (1):

$$2\pi r B = \frac{\mu_0 E_0}{\rho_0} \left[a^2 - \frac{2a^2}{\pi^2} + \frac{(b^2 - a^2)}{2} \right] 2\pi$$

$$B = \frac{\mu_0 E_0}{\rho_0} \left[a^2 - \frac{2a^2}{\pi^2} + \frac{(b^2 - a^2)}{2} \right] \frac{2\pi}{2\pi r}$$

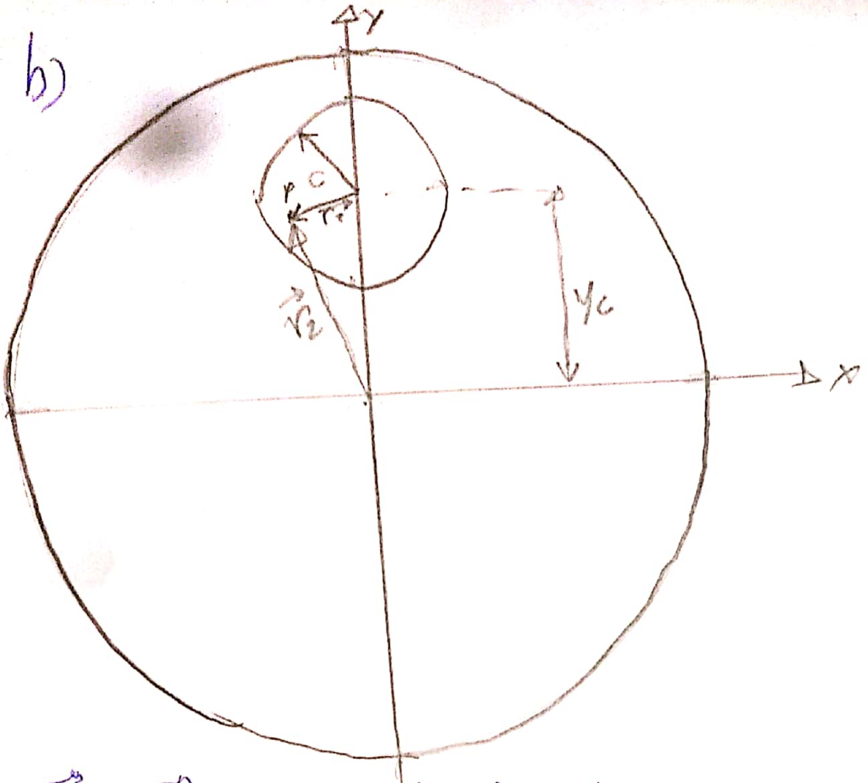
$$\vec{B} = \frac{\mu_0 E_0}{\rho_0} \left[\frac{a^2}{r} - \frac{2a^2}{\pi r} + \frac{(b^2 - a^2)}{2r} \right] \hat{\theta}$$

Ou com $I_{int} = I$:

$$B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B}(r) = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

b)



$\vec{y}_c + \vec{r_2} = r_1 \Rightarrow \vec{r_1} - \vec{r_2} = \vec{y_c}$

Campo magnético do condutor

$$dB_c = \frac{\mu_0 j r_1}{2}$$

na cavidade superior

$$dB_s = \frac{\mu_0 j r_2}{2}$$

Superposição dos campos do condutor e cavidade

$$dB_T = dB_c + dB_s$$

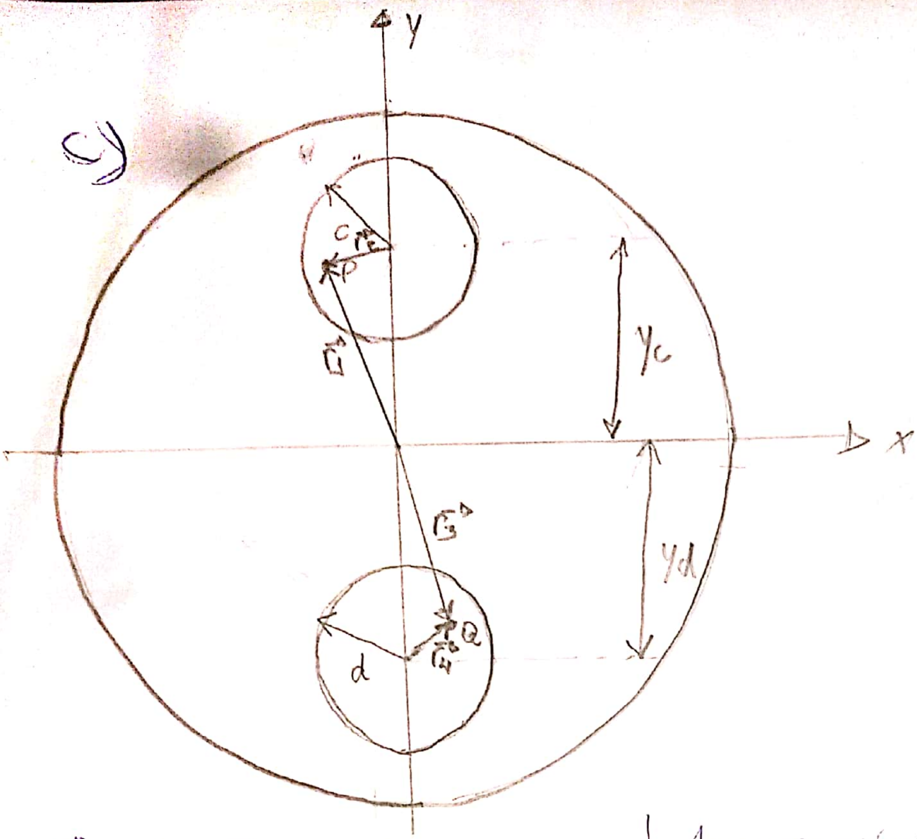
$$= \frac{\mu_0 j}{2} \times (r_1 - r_2)$$

(a densidade de carga é perpendicular aos vetores r_1 e r_2)

$$dB_T = \frac{\mu_0 j \vec{y}_c}{2}$$

$$B_T = \frac{\mu_0 y_c}{2}\left[\frac{(\pi^2 - 2^2)a^2}{\pi^2} + \frac{(b^2 a^2)}{2}\right]$$

c)



$\vec{y_c} + \vec{r_2} = \vec{r_1} \Rightarrow \vec{r_1} - \vec{r_2} = \vec{y_c}$

$\vec{y_d} + \vec{r_4} = \vec{r_3} \Rightarrow \vec{r_3} - \vec{r_4} = \vec{y_d}$

campo magnético no condutor - pela lei de Faraday

$$dB_c = \frac{\mu_0 j r_1}{2}$$

na cavidade superior

$$dB_s = \frac{\mu_0 j r_2}{2}$$

na cavidade inferior

$$dB_I = \frac{\mu_0 j r_3}{2}$$

Superpondo os campos do condutor e da cavidade superior e inferior

$$d\vec{B_T} = dB_c + dB_s + dB_I$$

$$= \frac{\mu_0 j}{2} \times [(r_1 - r_2) + (r_3 - r_4)]$$

(a densidade de carga é perpendicular aos vetores r_1, r_2, r_3 e r_4

$$dB_T = \frac{\mu_0 j}{2} \times [y_C + y_D]$$

$$B = \frac{\mu_0 I (y_C + y_D)}{2}$$

$$\frac{\mu_0 (y_C + y_D)}{2} \left[\frac{(\pi^2 2) a^2}{\pi^2} + \frac{(b^2 - a^2)}{2} \right]$$