2ª Prova de MAT211
Cálculo Diferencial e Integral III
20/05/2011

Nome : Gabarito
NºUSP :
Professor : **Jairo Zacarias Gonçalves**

Q	N
1	
2	
3	
Total	

1. (3,0 pontos) (i) Determinar os números naturais m e n para os quais o campo

$$F : I\!R^2 \to I\!R^2$$
$$F(x,y) = (3x^{m+1}y^{n+1})\, i + (2x^{m+2}y^n)\, j$$

é conservativo.

(ii) Com F de (i), calcule $\int_\gamma F \cdot d\gamma$, onde

$$\gamma : [0, 2\pi] \to I\!R^2$$
$$\gamma(t) = (2\text{cos}t)i + (\text{sen}t)j$$

(i) Seja $P(x,y) = 3x^{m+1}y^{n+1}$ e $Q(x,y) = 2x^{m+2}y^n$

Como P e Q estão definidas em $\mathbb{R}^2$ (que é conexo por poligonais em escadas) uma condição necessária e suficiente é que

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

$$2(m+2)x^{m+1}y^n = 3(n+1)\,x^{m+1}y^n \quad \text{, ou}$$

$$2(m+2) = 3(n+1)$$

$$m+2 = \frac{3}{2}(n+1)$$

$$\boxed{m = \frac{3}{2}(n+1) - 2} \quad \text{ou} \quad \boxed{m = \frac{3n-1}{2}}$$

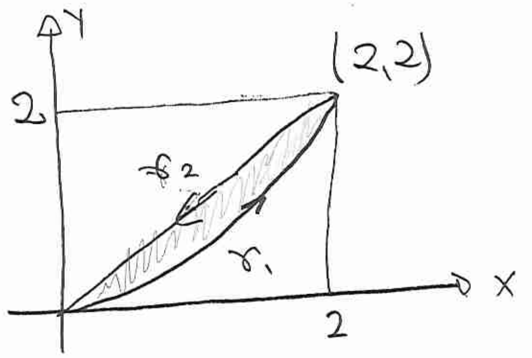
$\gamma(0) = \gamma(2\pi)$ e o campo é

2. (4,0 pontos) Determine a área da região limitada pela reta $y = x$ e pela curva $x = t^3 + t$ e $y = t^5 + t$ com $0 \leq t \leq 1$.

Como $0 \leq t \leq 1$

$t^5 \leq t^3 \Rightarrow t^5 + t \leq t^3 + t$

e $y \leq x$. Logo o gráfico da funções $x = t^3 + t$, $y = t^5 + t$ em $0 \leq t \leq 1$ é



$\gamma_1 : [0,1] \to \mathbb{R}^2$

$\gamma_1(t) = (\underbrace{t^3+t}_{X}, \underbrace{t^5+t}_{Y})$

$\gamma_2 = [0,2] \to \mathbb{R}^2$

$\gamma(t) = (\underbrace{2-t}_{X}, \underbrace{2-t}_{Y})$

$$A = \int_{\gamma_1 \cup \gamma_2} x\,dy = \int_0^1 (t^3+t)(5t^4+1)\,dt + \int_0^2 (2-t)(-1)\,dt$$

$$= \int_0^1 [5t^7 + t^3 + 5t^5 + t]\,dt + \int_0^2 (t-2)\,dt$$

$$= \left[5\frac{t^8}{8} + \frac{t^4}{4} + \frac{5t^6}{6} + \frac{t^2}{2}\right] + \left[\frac{t^2}{2} - 2t\right]_0^2$$

$$= \frac{5}{8} + \frac{1}{4} + \frac{5}{6} + \frac{1}{2} - 2 = \frac{15 + 6 + 20 + 12 - 48}{24}$$

$$= \frac{5}{24}$$

3. (3,0 pontos) Calcule a primeira derivada das funções dadas.

(i) $h(x) = \int_0^x sen(x^2t^2)dt$

(ii) $h(x) = \int_{x^2}^0 \frac{dt}{1+x^4t^4}$

(i) $\varphi(u,v) = \int_0^u sen(vt^2)\,dt$

$u = u(x) = x$

$v = v(x) = x^2$

$h(x) = \varphi(u(x), v(x))$

$h'(x) = \frac{\partial \varphi}{\partial u}\frac{du}{dx} + \frac{\partial \varphi}{\partial v}\frac{dv}{dx}$

$\frac{\partial \varphi}{\partial u} = sen(vu^2)$ $\qquad \frac{\partial \varphi}{\partial v} = \int_0^u \frac{\partial}{\partial v} sen(vt^2)\,dt$

$= \int_0^u t^2 \cos(vt^2)\,dt$

$$\boxed{h'(x) = sen(x^4) + 2x\int_0^x t^2\cos(x^2t^2)\,dt}$$

(ii) $h(x) = -\int_0^{x^2} \frac{dt}{1+x^4t^4}$

u $\qquad u$

$$u(x) = x^2 \qquad v(x) = x^4$$

$$h'(x) = \frac{\partial \varphi}{\partial u}(u(x), v(x))\, u'(x) + \frac{\partial \varphi}{\partial v}(u(x), v(x)) \cdot v'(x)$$

$$\frac{\partial \varphi}{\partial u}(u,v) = -\frac{1}{1+vu^4}$$

$$\frac{\partial \varphi}{\partial v}(u,v) = -\int_0^u (-1)(1+vt^4)^{-2}\, t^4\, dt = \int_0^u \frac{t^4\, dt}{(1+vt^4)^2}$$

$$h'(x) = -\frac{1(2x)}{1+x^4(x^2)^4} + \left[\int_0^{x^2} \frac{t^4\, dt}{(1+x^4t^4)^2}\right] 4x^3$$

$$\boxed{h'(x) = \frac{-1(2x)}{1+x^{12}} + 4x^3 \int_0^{x^2} \frac{t^4\, dt}{(1+x^4t^4)^2}}$$